THE DEVELOPMENT OF A MORPHOMETRIC MODEL FOR THE ESTIMATION OF MEAN ANNUAL SEDIMENT YIELD IN UNGAUGED CATCHMENTS: OF SOUTH AFRICAN RIVER SYSTEMS

'Submitted in fulfilment of the requirements for the degree of Master of Science in the Department of Geography, Rhodes University.'

by

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PREFACE

Hydrologists are regularly faced with the unenviable task of having to predict the magnitude and frequency of phenomena such as floods and droughts; and rates of erosion. If long records are available for analysis the hydrologist is able to base his predictions on the premise that the pattern of variation that has been observed in the past will persist in the future. The confidence that can be placed in any estimate consequently depends to a large extent on the length of time over which the phenomena have been measured at the problem site. Unfortunately the availability of adequate records tends to be the exception rather than the rule and in areas where there is inadequate data, it is necessary to resort to the hazardous procedure of transferring information from the gauged to the ungauged catchments. The transfer of information is accomplished by using empirical methods based on regionalised parameters, but the uncertainties involved together with the economic implications that could arise from a poor estimate, prompt the hydrologist to use as many methods as possible.

The need for empirical methods of estimating mean annual sediment yield in ungauged catchments was first appreciated by the author when he was involved in the estimation of design floods and sediment accumulation at sites for proposed reservoirs. Empirical methods of estimating sediment yield are frequently used in an engineering context, but little attention has been given to the catchment surface from which the sediment supply is derived. It is perhaps in this often neglected field of research that the physical geographer can make a contribution.

The principal aim of the thesis, more fully discussed in Chapter I, was the development of a morphometric model which could be used to estimate mean annual sediment yield in ungauged catchments in South Africa. The data used in the development of the model were drawn from the catchments, described in Appendix A, that cover a wide range of climate and topography.

A description of the approaches adopted by other researchers for the development of empirical models of estimating sediment yield which forms the background to the model has been included as Appendix B. The model was first developed in an elementary form as the focus of a research project which was documented in the form of three reports of research in progress (Roberts, 1973 a, b and c). Analysis of the pattern of variation of suspended sediment yield provided a better understanding of factors affecting sediment yield and supported the selection of the prediction variable (Horton's P ratio) which was used in the model. The concepts of network topology were utilised to gain insight into the environmental factors controlling both the P ratio and sediment yield. Reasons for the high correlation between the P ratio and sediment yield are suggested but it is felt that further research should be focused on this aspect.

In order not to break the continuity and development of the steps taken in the derivation of the model details of the calculations are collected in Appendices C, D and E. While many of the figures and tables presented in the thesis appeared in technical notes prepared entirely by the author for the Department of Water Affairs, the views expressed in the thesis do not in any way, either explicitly or by implication, represent any official view or policy of the Department of Water Affairs.

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CHAPTER I

INTRODUCTION

Characteristics of climate, topography and surface geology render most parts of South Africa naturally susceptible to erosion, resulting in high sediment loads in the river systems. The subsequent deposition of the sediment in reservoirs leads to an increase in the cost of developing the limited water resources in the Republic. In 1949 the Director of Irrigation reported that "... the loss due to siltation of storage space in reservoirs presents a serious problem in most parts of South Africa ... and in many cases the execution of storage projects has been deferred pending steps to reduce the silt menace " (Mackenzie, 1949, p 1). A high percentage of the sediment load brought into a reservoir is retained, displacing water storage capacity that has been created at great cost, and decreasing the economic life of the reservoir. The engineers and hydrologists responsible for the design of large water storage structures are consequently faced with the problem of estimating the rate of sediment accumulation so that it may be accommodated in the design capacity to ensure that the direct and indirect benefits expected from the project can fully redeem the capital outlay by the time storage becomes ineffective.

There are few catchment areas in South Africa where the movement of sediment has been recorded over a sufficiently long period of time to permit the derivation of reasonably accurate estimates of mean annual sediment yield. In most areas estimates are, of necessity, based on empirical methods which have been calibrated by using the scant sediment data available. There is often a large discrepancy between the estimates derived by using existing methods mainly because of the lack of data and the relatively low level of correlation between sediment yield and the prediction variables used. As a result it is unwise to use only one method for estimating mean annual sediment yield at any point in a river system. The most commonly used empirical methods of estimating mean annual sediment yield will be discussed in the following chapter.

The aim of the research project is to derive an empirical method of estimating mean annual sediment yield that can be used in conjunction with present methods to increase the level of confidence that can be placed in the chosen estimate at a problem site. The two main problems involved in the derivation of such a method are firstly the lack of data for calibrating a prediction model and secondly the lack of suitable prediction variables. The most reliable measurements of mean annual sediment yield are derived from regular trigonometric surveys of existing large reservoirs and the data derived in this way have been used for the calibration of the prediction model. Other data relating to the movement of sediment in river systems are available in the form of measurements of suspended sediment concentration and a discussion of the available data in the following chapter will show that the suspended sediment data are not suitable for the purpose of calibrating the prediction model. However, Leopold and Maddock (1953) have shown that measurements of suspended sediment concentration may, on the basis of certain assumptions, be used as an index to reflect relative variations of total sediment load. Selected records of suspended sediment concentration have been examined in Chapter III with the aim of identifying common patterns in the relative seasonal fluctuations of sediment load. The patterns of variation which emerged were then studied in terms of the seasonal variation in catchment surface conditions in order to see if they would provide a useful guide to the selection of prediction variables for the model. The choice of suitable prediction variables is the most important aspect of the model development, for a review of the available literature has indicated that, as yet, no single or group of easily measured variables has been found that can be used to explain a high proportion of the variation in sediment yields from one area to another. The dearth of suitable prediction variables may be attributed to the fact that the sediment yield of any river system is controlled by a very large number of variables[1], many of which are interdependent and difficult to measure in the field.

The examination of the relative fluctuations of sediment load in Chapter III led to the assumption that the problem presented by the lack of suitable prediction variables can best be approached by examining the relationship between sediment yield and easily measured catchment variables that do not necessarily have a direct influence on the sediment yield but vary in sympathy with it from one area to another. In Chapter IV the concept of a drainage basin as an open system in steady state has been used to show that there is a conceptual link between the morphology of a channel network and the mean annual sediment yield of the system. The variable chosen for the prediction of sediment yield was a morphological network index, the P ratio, developed by Horton (1945), and its link with the sediment yield is indirect in the sense that both the sediment yield and the chosen variable are functions of the same set of environmental factors and should vary in parallel from one area to another. The link between network morphology and sediment yield has led to the formulation of the hypothesis that at any point in a river system there is a relationship between the mean annual sediment yield per unit of drainage area and Horton's P ratio of the channel network above that point, provided the watershed system conforms to the following conditions:

(i) steady state conditions prevail in the drainage basin, and

(ii) Horton's two laws of stream lengths and stream numbers hold true for the drainage network.

The term watershed system refers to the open system encompassed by the watershed of the drainage basin. Those aspects of network morphometry that are related to the derivation and interpretation of Horton's P ratio will be discussed in Chapter V and the hypothesis will be tested in Chapter VI.

The model for the estimation of mean annual sediment yield in ungauged catchments has been based on the above hypothesis. Consequently the efficient operation of the model is subject to compliance with the conditions stipulated in the hypothesis. The lack of methods for testing the first condition, namely that of steady state, has led to an application of network topology (Chapter VI) in which a new measure of the topological structure of dendritic drainage networks (the E index) has been used. The E index, developed by Jarvis (1972), can be interpreted in terms of network growth processes and it is this aspect of the topological index that has been used to draw inferences about steady state conditions in those drainage basins examined during the testing of the hypothesis.

When formulating a prediction model it is often wise to reserve some of the available data for the subsequent testing of the performance of the model. In the case of the model for the prediction of mean annual sediment yield, the available data is so limited that the reservation of some of the data would seriously detract from the performance of the model. All the available data will therefore be used for the formulation of the model and the performance of the model will be assessed in Chapter VII by applying it to the St Lucia Lake system. The mean annual sediment yield of the St Lucia system has not been measured but the problems caused by sediment accumulation have resulted in detailed research by a number of investigators in recent years and their estimates of the sediment yield will form a basis for comparison with estimates derived from the model.

As the quality and quantity of sediment data form an important consideration in the selection of prediction variables and in the calibration of the model, the sediment data available in South Africa have been assessed in Chapter II before proceeding with the formulation of the model. A discussion of existing empirical methods of estimating sediment yield has been included in the following chapter as the prediction variables used in these methods are of relevance to the choice of prediction variables for the model.

CHAPTER II

AN ASSESSMENT OF THE AVAILABLE DATA

The calibration of a model for the estimation of mean annual sediment yield in ungauged catchments requires the use of data on the sediment load carried by rivers over a long period of time (preferably more than fifteen years) and the data should cover a wide range of South African climatic and geological conditions. The spatial and temporal prerequisites of the data preclude personal field measurement so that the data must, of necessity, be obtained from literature and government agencies responsible for the collection of sediment data. The relevant data are collected by the Division of Hydrology, Department of Water Affairs, from their network monitoring water quality and quantity throughout the country. Sediment data collected by other agencies are often sent to the Division of Hydrology with the result that the Division has the most comprehensive sediment data bank that is readily available to the research worker. Consequently most of the data used in the analysis have been drawn from records at the Division of Hydrology.

The sediment yield of a river network can be defined as the total sediment load transported by the network per unit of time. The total sediment load is usually[2] subdivided into two categories:

(i) the suspended sediment load which is the sediment held in suspension either by the upward components of turbulence or by colloidal suspension, and

(ii) the bed load which comprises the sediment that is too heavy to be carried in suspension and is moved downstream by saltation and rolling.

The distinction between the two categories is seldom clear since the proportion of the total load carried in suspension at any time at any point in a river channel is dependent to a large extent on the velocity of the water and the amount of turbulence created by channel configuration and bed roughness. Since the sediment yield of a river is the total sediment load transported per unit of time, measurement of both the suspended and bed load is necessary for estimates of the sediment yield. The necessity of measuring the total sediment load for estimates of sediment yield is one of the most important criteria in the assessment of the available data.

The available sediment data in South Africa can be subdivided conveniently into two distinct types. The first type is the reservoir survey data which takes the form of measurements of total sediment load and is derived from regular trigonometric surveys of existing large reservoirs. These measurements are the most reliable measurements of mean annual sediment yield and they will be used for the calibration of the model. The second type of data takes the form of measurements of suspended sediment concentrations in rivers. An assessment of the suspended sediment data will show that the data have limitations which preclude their use for deriving estimates of mean annual sediment yield. However, the data will be used to illustrate relative fluctuations in sediment load and will serve as a guide to the selection of prediction variables for the model. The assessment of the sediment data available will also facilitate an appraisal of present empirical methods of estimating mean annual sediment yield in South Africa.

Reservoir survey data

The data obtained from trigonometric surveys of reservoirs provide the most reliable estimates of mean annual sediment yield. Most large reservoirs in the country are surveyed at intervals of between two and five years in order to determine the change in maximum storage volume (often referred to as full supply capacity) due to sediment accumulation in the reservoir basin since the previous survey. A knowledge of the maximum storage volume is essential for the efficient control and delimitation of water usage from the reservoir. The surveys also provide a means of monitoring the sediment yield of the river as the difference in storage volume between the two survey periods represents the total volume of sediment that has accumulated in the intervening period.

River water flowing into a reservoir experiences a sharp reduction in velocity with the result that a large proportion of the suspended sediment load is deposited in the reservoir together with the bed load and the accumulating volume represents the total sediment load of the river. The percentage of the total sediment load retained by a reservoir over a long period is referred to as the 'trap efficiency' of the reservoir and it is generally accepted that reservoirs with a storage capacity in excess of the mean annual runoff of the contributing rivers have a trap efficiency of 100%. Sediment yields derived from reservoirs with a storage capacity smaller than the mean annual runoff have been adjusted by using a trap efficiency index to compensate for loss of sediment.

Prior to 1972, the reservoir surveys were conducted when the level of the water in the reservoir was as low as possible so that most of the survey could be conducted in dry conditions by normal trigonometric methods. Those portions of the reservoir basin that were still under water were surveyed by depth ranging from a boat. However, with the introduction of the 'echo-sounder' survey equipment at the Department of Water Affairs in 1972, the surveys are conducted when the reservoirs are as full as possible since the entire survey can be done accurately from the boat. The difference in the water level for the two survey techniques could give rise to a difference in the volume occupied by a unit weight of sediment. In order to avoid non-homogeneity in the data it has been decided to exclude from the analysis data acquired after 1972.

There are fifteen large reservoirs in South Africa[3] where surveys have been conducted over a period in excess of fifteen years. The reservoirs are listed in Table 1. The values of mean annual sediment yield derived from these reservoir surveys have been used for the calibration of the model for the estimation of mean annual sediment yield in ungauged catchments.

TABLE 1

MEAN ANNUAL SEDIMENT YIELDS AND P RATIOS FOR SELECTED DRAINAGE SYSTEMS.

Reservoir System	Reservoir No.	C % MAR 1	TECI 2	AMAS 3	MAS 4	rb 5	rl 6	P 7
Van Ryneveld's Pass	N1R01	165	-	0,303	0,303	4,00	2,42	0,605
Lake Arthur	Q4R01	87	1,3	0,605	0,595	3,66	3,54	0,967
Grass Ridge	Q1R01	143	-	0,412	0,412	3,03	2,55	0,840
Lake Mentz	N2R01	82	1,8	0,276	0,270	4,25	2,48	0,583
Tierpoort	C5R01	169	-	0,156	0,156	4,00	2,60	0,650
Prinsrivier	J1R01	45	5,5	0,088	0,085	4,00	2,33	0,583
Kammanassie	J3R01	79	2,3	0,103	0,101	3,70	2,30	0,622
Koppies	C7R01	8	9,2	0,198	0,182	3,84	2,69	0,700
Olifants Nek	A2R03	111	-	0,089	0,089	3,50	2,23	0,637
Schweitze Reneke	C3R01	26	7,4	0,009	0,008	4,91	2,75	0,560
Egmont	D2R01	86	1,4	0,260	0,257	4,00	2,84	0,710
Kommando Drift	Q4R02	156	-	0,210	0,210	4,14	2,82	0,681
Oukloof	J2R03	65	3,5	0,038	0,036	4,63	2,52	0,544
Albasini	A9R01	64	3,6	0,135	0,130	3,51	2,47	0,641
Loskop	B3R02	38	6,2	0,011	0,010	3,85	2,16	0,560

1) Reservoir capacity as percentage of mean annual runoff.

2) Trap efficiency correction index as a percentage.

3) Adjusted mean annual sediment per sq. km. $(10^3 m^3)$.

4) Mean annual sediment per sq. km. $(10^3 m^3)$.

5) Bifurcation ratio.

6) Length ratio.

7) P ratio.

TABLE 2

SELECTED RECORDS OF SUSPENDED SEDIMENT CONCENTRATION

Code	Place	River	Max. Rainfall
Q4M02	Roberts Kraal	Vlekpoort	Summer
V1M01	Colenso	Tugela	Summer
D1M03	Aliwal North	Orange	Summer
D7M05	Upington	Orange	Summer
H5M02	Wolvendrift Annex	Breë	Winter
C3M03	Taung	Hartz	Summer
U1M04	Research Station	Mkomaas	Summer

Suspended sediment data

The suspended sediment concentration at any point in a river at any moment in time can be estimated by analysing a sample of the river water. A number of sampling stations have been established by the Department of Water Affairs and the samples are analysed in Pretoria to determine the concentration (expressed as a percentage by weight or volume) of suspended sediment. The samples are taken by the 'gulp sampling method' which entails dipping a container into the water to take an instantaneous sample near the surface that will contain suspended sediment with little or none of the saltating load. As there is usually a variation of concentration of suspended sediment with depth as well as a variation across the stream section (the velocity of the water is seldom uniform from one bank to another) the gulp samples represent the concentration of suspended sediment at only one point in a river cross-section and are not necessarily representative of the mean concentration in the horizontal or vertical planes at the time of sampling.

It is becoming common practice in other countries (Australian Water Resources Council, 1969) to improve the sampling technique by taking at least four samples spaced across the river to account for horizontal variation and to use 'depth-integrating' samplers to overcome the problem of depth variation. The depth-integrating samplers have a constricted inlet to the sampling container which allows a constant stream of water into the container as it is lowered at a uniform rate from the surface to a point approximately thirty centimetres above the river bed. As a result, the container holds water taken from a range of depths and the measured concentration is more representative of the mean concentration at the place of sampling. Unfortunately very few depth-integrated samples have been taken in South Africa.

While the available measurements of suspended sediment concentration are probably not representative of the average concentration at the time of sampling even greater deviations from the true total suspended load may result from the necessary assumption that the concentration measured remains constant until the time of the next sample. Suspended sediment concentration is characteristically very variable with time, and attempts to estimate the pattern of change of concentration between samples by reference to fluctuation of discharge have met with little success. Middleton and Oliff (1961, p 241) found in their study of suspended sediment in the Tugela River that, "The concentrations of suspended silt proved to be most variable, not only in relation to wet and dry seasons, but also in relation to similar stages of flow. Concentrations bear no relationship to stage except at the extreme ranges of the latter when the relationship is only general." They found wide variations of concentration from hour to hour and give an example at Bergville on the Tugela River where "concentrations varied from 0,2 percent to 2,78 percent on two consecutive days, the higher value being the maximum recorded at this station " (Middleton and Oliff, 1961, p 241).

The complete absence of any direct relationship between suspended sediment concentration and stage (the level of the water in the channel) of a river is usually attributed to the fact that there may be more loose material ready to be removed from the catchment surface at the beginning of a storm than at the end of a storm, and that the catchment surface conditions for a winter storm may be entirely different from those for a summer storm. The erratic supply of sediment is aggravated by some tributaries producing higher concentrations than others which makes the sediment supply dependent on the storm centre. The characteristic variability of suspended sediment concentration gives rise to the necessity of having long records of frequent measurements of suspended sediment concentration in order to estimate the mean annual suspended sediment concentration.

The measurements of suspended sediment concentration contain no measurement of the bed load fraction with the result that some estimate of the bed load must be added to estimates of mean annual suspended sediment concentration in order to obtain estimates of mean annual sediment yield. Unfortunately, measurements of the bed load are very seldom available mainly because no satisfactory method of measuring bed load has yet been devised. As pointed out by Leopold and Maddock (1953, p 20), "Any sampler placed on the stream bed provides an obstruction to the natural flow and tends to set up turbulent eddies in the immediate vicinity which so change the transport near the bed that the resulting sample becomes nonrepresentative." Consequently some arbitrarily chosen volume of bed load sediment must be added to the volume of suspended sediment in order to obtain the total sediment load.

Most of the available records of suspended sediment concentration are either disjointed or have a sample frequency that is too large for reasonable interpolation between measurements. However, seven of the records of suspended sediment at the Department of Water Affairs were found to contain reasonably continuous daily observations over a number of years together with concomitant records of discharge and stage (Table 2). The records for the places listed in Table 2 are not considered adequate for deriving estimates of mean annual sediment yield because of the characteristic variability of suspended sediment concentration, the lack of bed load measurement and because of discontinuities in the records. Although the suspended sediment records have limitations for model calibration they are, subject to an assumption, a valuable source of information. The average ratio of bed load to suspended sediment load is known to vary from one river section to another, but if it is assumed that the ratio remains reasonably constant at a particular sampling point, the records of suspended sediment concentration can be regarded as indicative of fluctuations of sediment load. The seven records of suspended sediment concentration will therefore be examined for common periodicities since such periodicities, if found, will provide a valuable guide to the understanding of conditions controlling sediment yield and a guide to the selection of suitable prediction variables for the model.

Before analysing the data available, present empirical methods of estimating sediment yield will be briefly discussed with particular reference to the data and the prediction variables used.

Present empirical methods[4]

There are three empirical methods of estimating mean annual sediment yield that have been used in South Africa. The first guide to the estimation of sediment yield took the form[5] of a silt map compiled by Midgley (1952) which depicted long term silt movement in South Africa. The map is part of an unpublished thesis but copies of the map itself have become available, and for many years the map provided the only method of readily obtaining estimates of mean annual sediment yield for any catchment in South Africa. An alternative method was produced by Schwartz and Pullen (1966) who made use of additional data that had become available since the formulation of Midgley's map. The method of Schwartz and Pullen was based on the observation that the range of sediment production tends to narrow with increase in catchment size. Mean annual sediment yields per unit area derived from reservoir surveys were plotted against catchment area. Converging trend lines were then drawn in where the trend lines represented indices of silt production. Interpolated index values related to the results of the reservoir sediment surveys were plotted on the map at their appropriate geographical locations and five zones of equal index of silt production were delimited according to the index values. The area covered by each zone is necessarily extensive with the result that the method provides a wide range of possible sediment yields for any one catchment and the range increases with decreasing catchment area.

A method of estimating mean annual suspended sediment yield in South Africa was produced by Doornkamp and Tyson (1973). The method takes the[6] form of maps depicting the variation of suspended sediment yield where the yields have been derived by application of the Fournier (1960) equation;

$$E = \frac{(p^2 \ / \ P)^{2,65} . \ Cm^{0,46}}{1,56} \qquad \text{(Eqn 1)}$$

where E is the mean suspended sediment yield (tons km^{-2} $annum^{-1}$), p is the maximum mean monthly rainfall (mm), P is the mean annual rainfall (mm) and Cm is defined by H tan Ø with H being mean basin height (m) and Ø mean basin slope. The values of mean annual suspended sediment yield could be used to estimate mean annual sediment yield if some intuitively derived estimate of bed load is added but "it must be stressed, however that point interpolation of suspended sediment yields is unwise owing to the fact that in particular areas local controls may exert a greater influence on yields than those of precipitation and relief" (Doornkamp and Tyson, 1973, p 340).

The prediction variables used in the above models have been catchment area and the characteristics of rainfall and relief. The variables have been used as representative of the complex interaction of the catchment surface characteristics that control the amount of sediment reaching the stream channels. As the suspended sediment supply is essentially independent of the channel system and is derived from the catchment surface, inferences about the interaction of catchment surface characteristics will be drawn from the examination of the fluctuation of suspended sediment concentration in the following chapter. A better knowledge of the controlling catchment conditions may provide a guide to the choice of a prediction variable that will explain a greater proportion of the variation in sediment yields than explained by the variables used in the above methods.

CHAPTER III

SEASONAL VARIATION OF SEDIMENT YIELD

It was reasoned in Chapter II that the variations in the sediment load supplied to a river channel are primarily the result of variations in catchment surface conditions. Consequently the search for suitable prediction variables should be directed to the variations of catchment surface conditions and the variables chosen should be an index of the complex interaction of all the catchment surface variables involved. Inferences about the relative variation of catchment surface conditions can be drawn from an examination of the relative fluctuations in sediment load. Leopold and Maddock (1953) have shown that suspended sediment load can be used as an index of total sediment load, and therefore the seven records of suspended sediment concentration listed in Table 2 may be used for this purpose. Should common patterns of variation during the year be found in the records of suspended sediment concentration, the patterns would provide a valuable guide to the understanding of the dominant factors controlling sediment yield. Nevertheless, it should be stressed that as the measurements of suspended sediment concentration are to be used as an index of total sediment load, only the patterns of variation in sediment concentration are of relevance and the individual values[7] used should be considered as being dimensionless.

The method of analysis

An examination of the records of suspended sediment concentration indicate that one month is the smallest time increment that may be used for reasonable estimation of mean concentration and yet be small enough to reflect at least seasonal variation during the year. The mean daily concentration was calculated for each month of the record at each station. These values were then averaged for each month of the year to obtain the long term mean daily concentration for each month of the year at each station. The average values for each month of the year were then plotted as an index of sediment load to display the average seasonal pattern of variation.

In six of the seven records the average pattern of seasonal variation in sediment load was characterised by a clear double peak during the year. The values obtained for Station U1M04 on the Mkomaas River have been plotted in Figure 1 as representative of the pattern that emerged in the six records. The only station that did not have a clear double peak was Station D7M05 on the Orange River where the pattern was characterised by a triple peak (Figure 2). An examination of the mean daily concentrations for each month in any one particular year at all stations showed that the double peak is usually, but not always present in any one year and that the double peak, if present, is not always in phase with the double peak of the average long term pattern of variation. As a result the average long term pattern of variation (Figure 1) is a suppressed form of a double peak pattern that usually occurs in any one year. The double peak phenomenon is also prominent in mean daily suspended sediment concentrations for each month for the Vaal River published by McCrae (1945) and for the Tugela River published by Middleton and Oliff (1961).

High discharges in rivers are usually associated with above average suspended sediment concentrations which led to the supposition that a double peak in the sediment load would be the result of a double peak in the discharge hydrograph. Yet, when the mean daily flow for each month of the year was calculated for each station over the same periods as the suspended sediment records, it was found that there was a single peak in the flow curve at all stations. Figure 1, which represents the pattern found at six of the stations, shows a double peak in the sediment load where the second higher peak coincides with the single peak of the mean discharge curve. On the other hand, Figure 2 for Station D7M05 shows a triple peak in the sediment load with the highest peak occurring after the period of maximum discharge. The significance of the triple peak at Station D7M05 will be

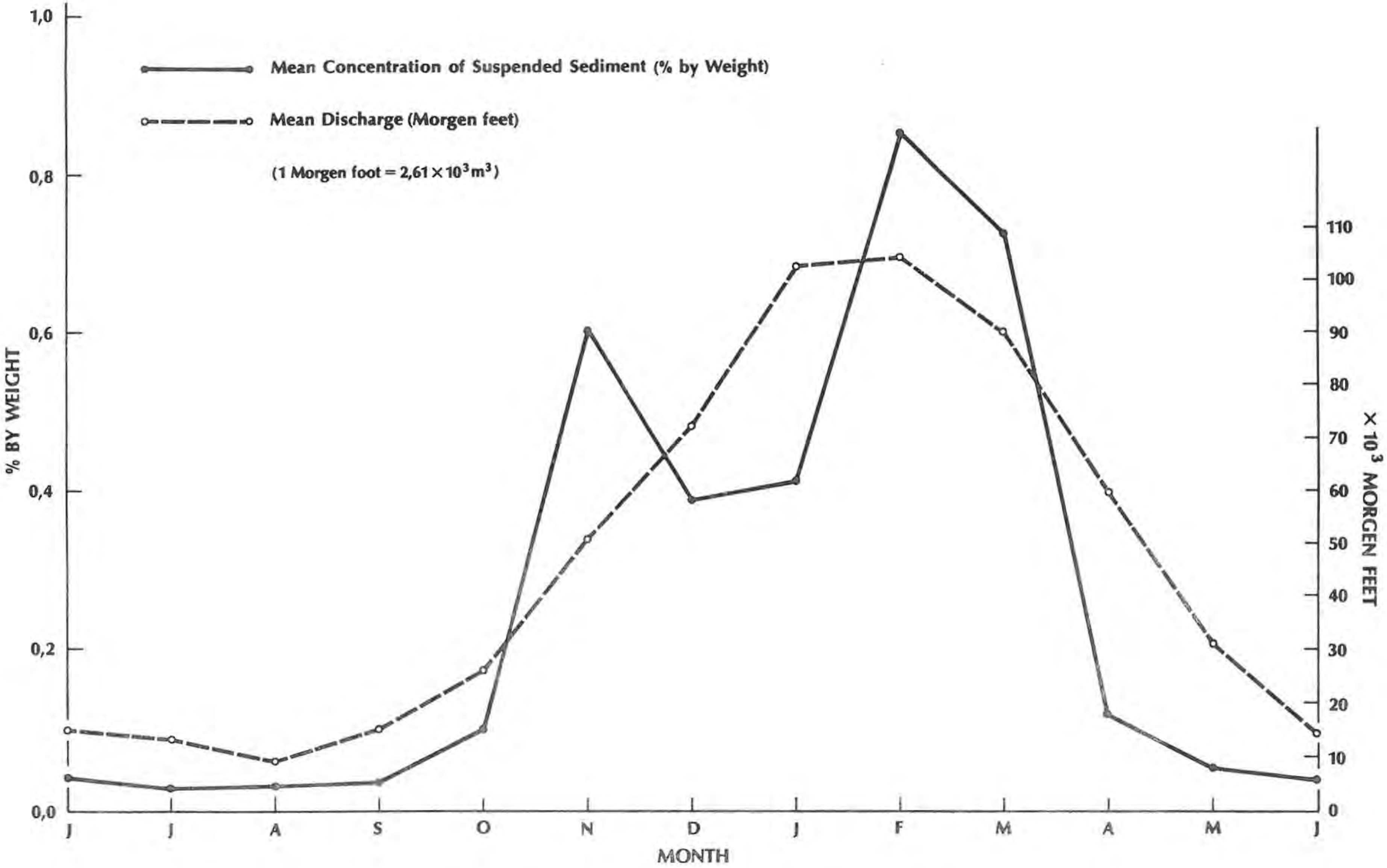


Figure 1 Seasonal Variation of Sediment Load;
Mkomaas River at Station U1M04

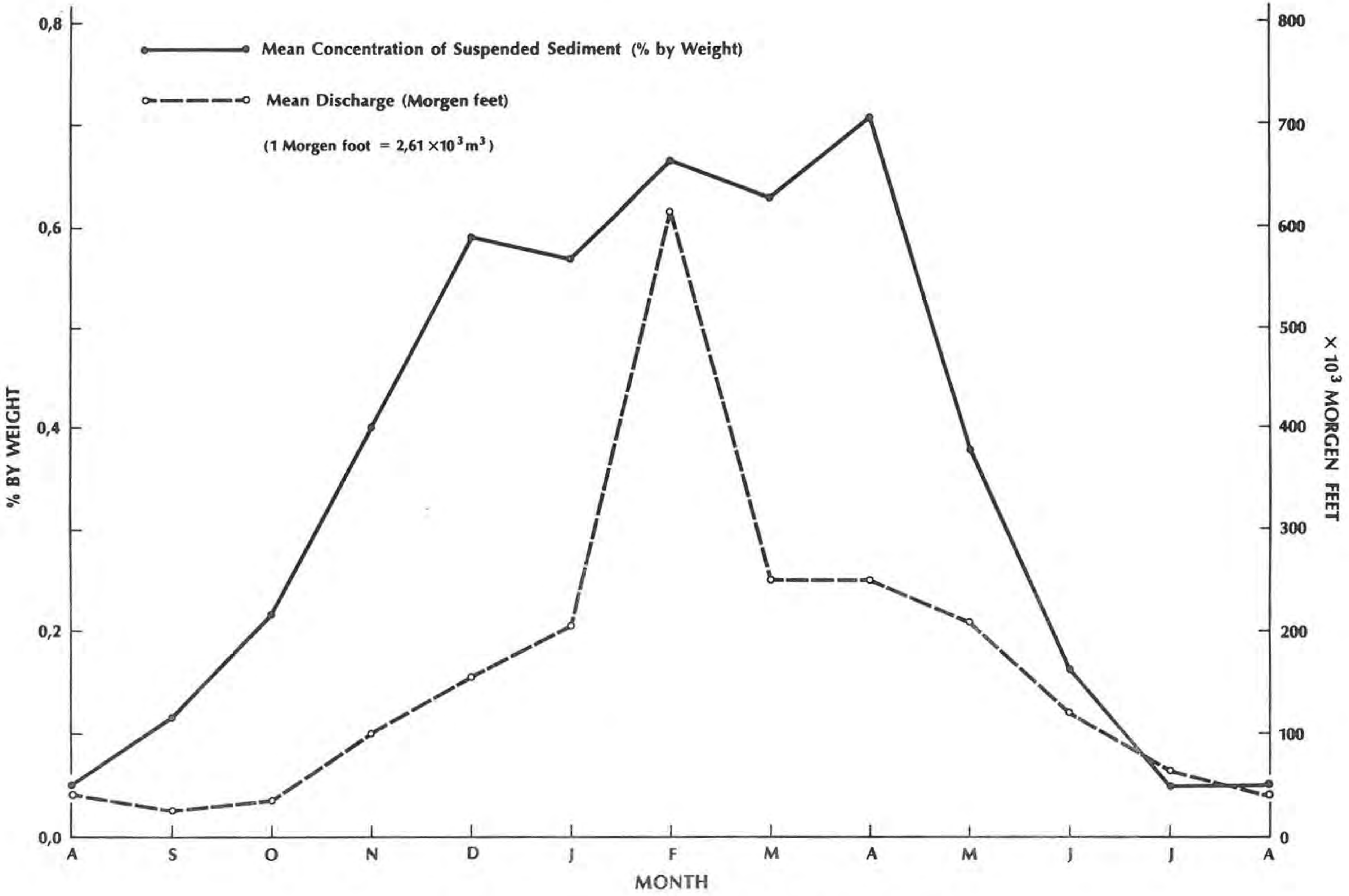


Figure 2 Seasonal Variation of Sediment Load;
Orange River at Station D7M05

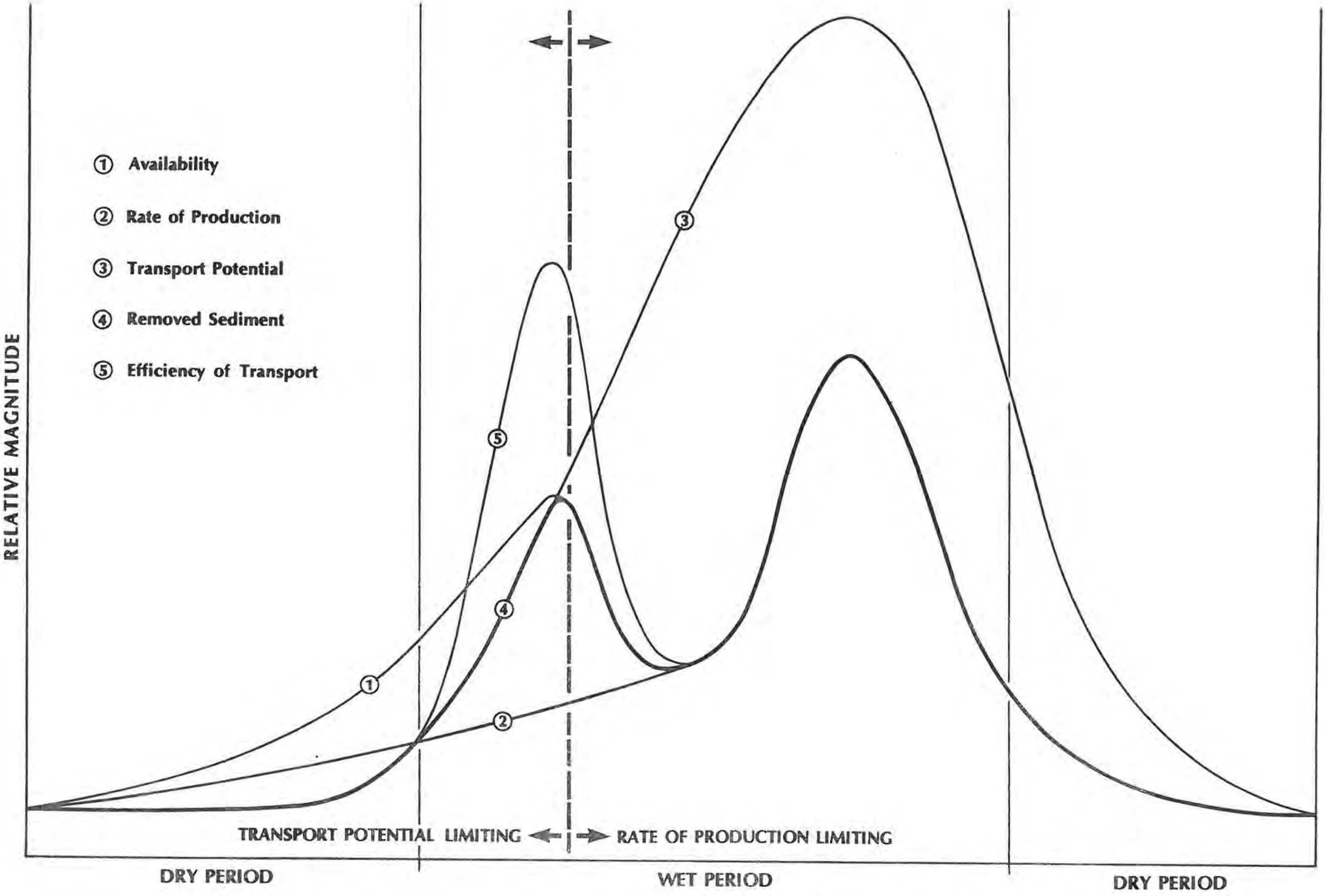


Figure 3 Seasonal Variation of Catchment Surface Conditions that Control Sediment Load

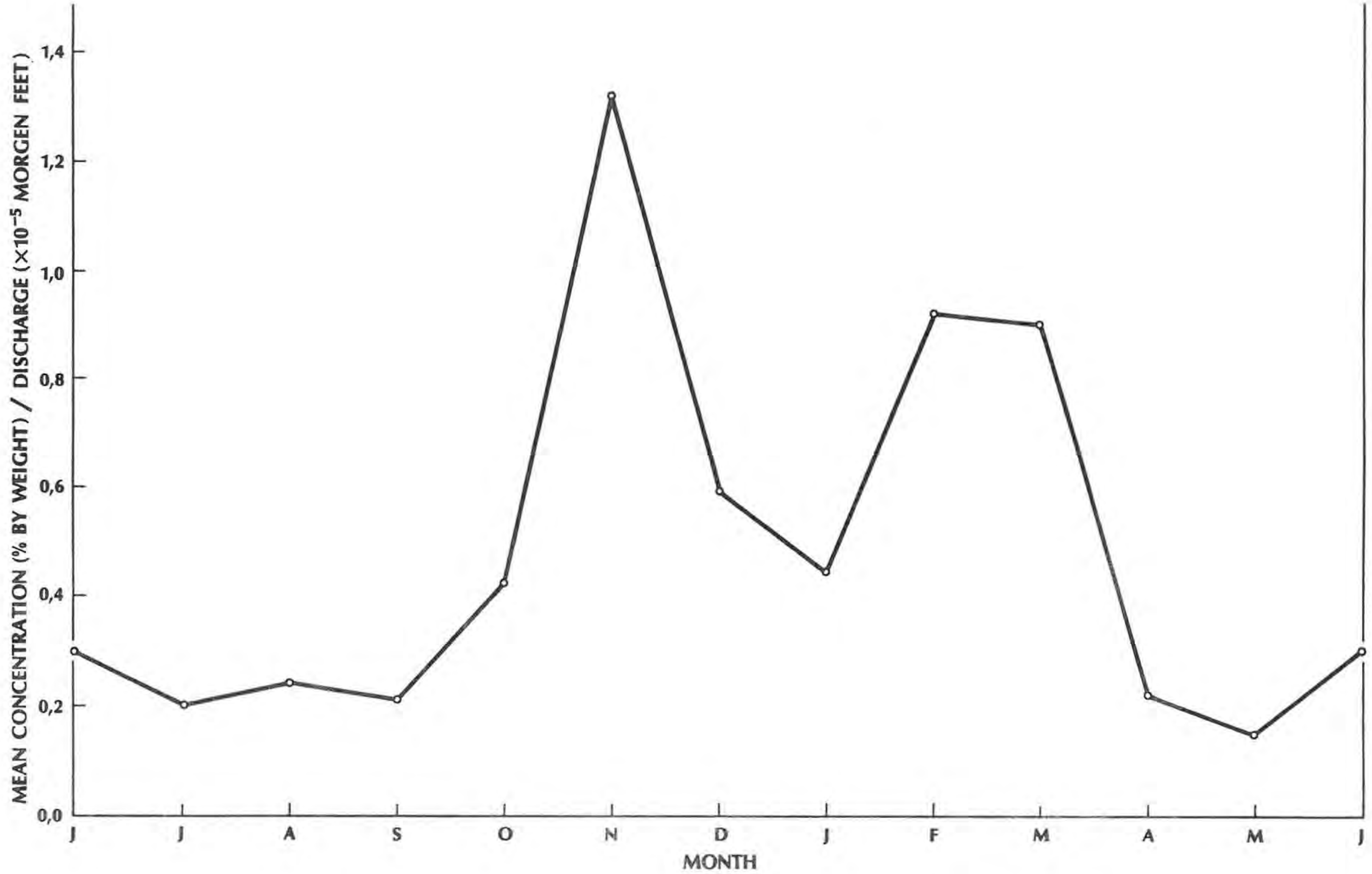


Figure 4 Seasonal Variation of the Efficiency of Transport Index for the Mkomaas Catchment above Station U1M04

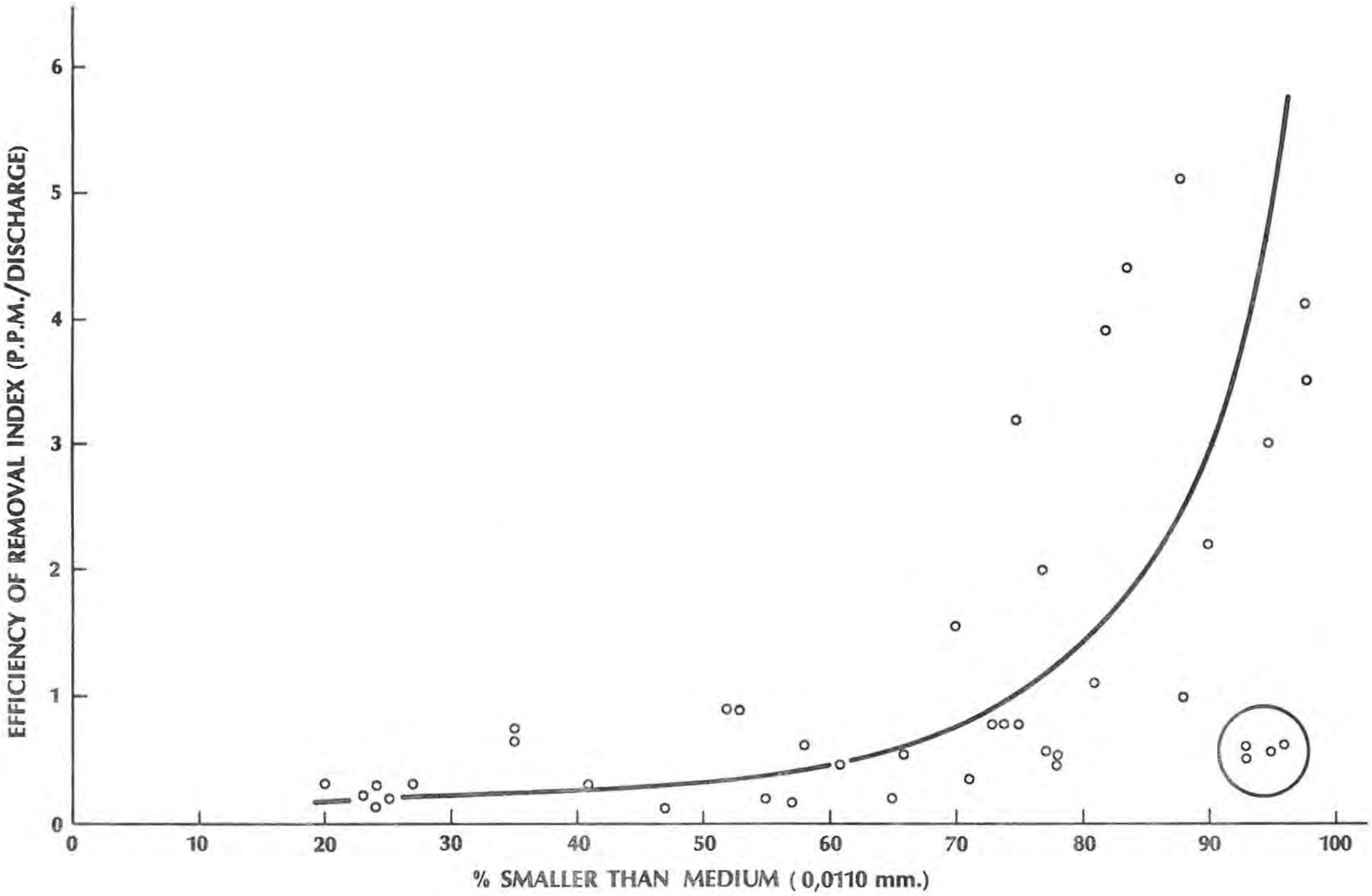


Figure 5 Particle Size Distribution of Suspended Sediment Samples for the Orange River at Station D3M11

discussed subsequent to an explanation of factors that give rise to the double peak observed at the other stations.

Although only seven records of suspended sediment concentration have been examined in detail it is assumed that the double peak phenomenon is the general pattern of variation in areas where the distribution of rainfall is seasonal, giving rise to a relatively wet and a relatively dry period during the year. The wet period may occur in either winter or summer as illustrated by the example of the Breë River (Table 2). Considering that the majority of the sediment load is supplied to the river channels from the catchment surface by surface runoff (overland flow), seasonal variations of sediment load can best be explained in terms of seasonal variation in catchment surface characteristics. In order to facilitate this explanation the terminology discussed below has been introduced by the author to describe the catchment surface variables involved. The relevant terms are:-

(i) The removed sediment is the quantity of sediment passing the gauging point at any time.

(ii) The transport potential is the volume of overland flow available at any time to transport the detached sediment particles to the stream channels. It should be noted that this is not intended to be a measure of the energy available for detaching sediment particles.

(iii) The rate of production of sediment is the quantity of sediment made available for transportation, per unit of time, by the erosive scour of overland flow on the catchment surface and by the processes of mechanical and chemical weathering.

(iv) The availability of sediment is the quantity of detached material on the surface of the catchment that can be readily transported to the stream channels by overland flow.

(v) The efficiency of transport is a measure of the extent to which each unit volume of water is acting as a transporting medium for sediment and is given by the concentration of sediment per unit volume of flow at the gauging point.

The explanation of the double peak phenomenon

It is reasoned that the sediment load in a river channel at any time is controlled by the mutual interaction of the above catchment variables. The rate of production and transport potential may be regarded as the principal variables because the sediment load is limited by one of these variables at any time. The results obtained from the Mkomaas River at Station U1M04 (Figure 1) will be used to derive the shapes of the curves in Figure 3 where the curves represent the variation of the defined variables during the year. The shape of each of the curves and the interdependence of the variables will be explained with the aid of Figure 3 to develop a composite pattern of variation that gives rise to the double peak phenomenon.

i) The removed sediment curve.

The removed sediment curve (Figure 3) represents the variation in sediment load during the year and has been given the same general shape as the sediment load curve in Figure 1 on the assumption that suspended sediment concentration can be used as an index of total sediment load.

ii) The transport potential curve.

The transport potential curve (Figure 3) has been given the same shape as the mean flow curve in Figure 1 since the single peak (in the wet season) represents the relative increase in overland flow as compared to the predominant base flow (low transport efficiency) during the dry months.

The first peak in the sediment load index in Figure 1 coincides with the rapid increase in the mean flow curve at the beginning of the wet season and as both sediment load and runoff are low during the dry months it can be assumed that sediment removal is limited by transport potential during the dry months. The transport potential curve (Figure 3) has therefore been drawn to coincide with the removed sediment curve during the dry period but lies above it during the wet period when transport potential no longer limits the removal of sediment.

iii) The rate of production curve.

The rate of production of sediment limits the removal of sediment during the wet period when the transport potential is high. The rate of production curve has been drawn accordingly so that it coincides with the removed sediment curve during the wet period. The decrease in the removal of sediment after the first peak in the removed sediment curve without a corresponding drop in transport potential (Figure 3) suggests that not all the sediment being produced during the dry period was being removed. If this were not the case, the decrease in the removal of sediment would be associated with a sharp, short term decrease in the rate of production of sediment which, in view of the steadily increasing transport potential seems unlikely. As a result of the conclusion that the sediment produced during the dry period was not being removed at the same rate at which it was being produced, the rate of production of sediment curve, during the dry period, was placed above the transport potential curve so that the height of the rate of production curve increases as the dry season progresses.

iv) The availability curve.

As the rate of production is greater than the transport potential during the dry period not all the sediment being produced is removed with the result that sediment accumulates on the catchment surface. The availability curve was drawn in accordingly so that during any month in the dry period it lies at a height above the rate of production curve equal to the sum of the differences between the rate of production and transport potential curves for the previous dry months. That is, its height above the rate of production curve at any time in the dry period represents the amount of sediment that has accumulated on the catchment surface. However, when the transport potential becomes greater than the rate of production at the beginning of the wet season all the accumulated sediment is removed and the availability curve drops to join the rate of production curve since the availability is limited by the rate of production during the wet season.

v) The efficiency of transport curve.

The shape of the efficiency of transport curve was determined by plotting mean concentration of suspended sediment divided by mean discharge against time for Station U1M04 on the Mkomaas River (Figure 1). From Figure 4 it can be seen that the efficiency of transport curve has the same shape as the removed sediment curve except that the first peak is higher than the second peak. During the wet period, when the transport potential is high, the sediment is removed at the optimum rate and the efficiency of transport curve represents the interrelationship between transport potential and availability. The first peak in the efficiency of transport curve coincides with a period of high availability and a transport potential factor that is low but sufficient to remove the sediment at the optimum rate. The result is that the concentration of sediment per unit volume of flow is high. However, when the excess sediment has been removed there is a drop in the efficiency of transport due to a drop in availability, coupled with continued increase in transport potential. The efficiency of transport is now limited by the rate of production (Figure 3) and rises to the second peak with increase in rate of production and the associated increase in availability. During the dry period the transport potential is too low to remove the sediment at the optimum rate and consequently the efficiency of transport is limited by the transport potential.

Particle size analysis

The rate of production has been defined as the quantity of sediment made available for transportation to the stream channels by erosive scour on the surface of the catchment and by the processes of chemical and mechanical weathering. Because sediment accumulates on the catchment during the dry periods when there is little overland flow, it is reasonable to assume that the weathering component of the production rate is dominant[8] during this period and that the erosive scour component is dominant during the wet period when transport potential exceeds the rate of production. The implication is that sediment removed at the beginning of the wet period is predominantly the product of weathering and is likely to be finer in texture than sediment removed during the periods of high flow when erosive scour is dominant. The relationship is somewhat obscured by an increase in the rate of chemical weathering during the wet period if the wet period is in summer. The efficiency of transport has been shown to be highest during the removal of accumulated sediment so that a plot of efficiency of transport against particle size should show that particle size tends to be small when efficiency of transport is high. The relationship would be partially obscured by the second, smaller peak in the efficiency of transport curve that occurs at the height of the wet season when erosive scour is dominant. Details of particle size in some suspended sediment samples are kept at the Department of Water Affairs, and although the data are inadequate for a detailed examination of the relationship between particle size and efficiency of transport, the data should serve to illustrate the general relationship.

A three year record of suspended sediment samples (as far as can be ascertained these samples are the only depth-intergrated samples taken in South Africa) were taken in the Orange River prior to the construction of the Hendrik Verwoerd Dam. These samples were analysed by the Department of Water Affairs to determine the range of particle size in each sample. For each sample the Department of Water Affairs have drawn cumulative frequency diagrams showing particle size plotted against the percentage of particles in the sample that are smaller. These diagrams have not been published. The relative change in particle size with change in efficiency of transport may be illustrated by plotting the percentage of particles smaller than any one chosen particle size against a trap efficiency index for each sample. For the purpose of this study it was decided to choose the medium silt size (0,0110 mm) as a standard where the medium size has been drawn from the following classification of suspended silt sizes used by the Department of Water Affairs;

Less than 0,0047 mm	fine silt
0,0047 - 0,0204 mm	medium silt
0,0204 - 0,0600 mm	coarse silt
greater than 0,0600 mm	fine sand

The silt size of 0,0110 mm was a convenient size to choose within the medium silt class. The percentage of particles smaller than medium silt size for each sample was read off the relevant cumulative frequency diagram and plotted against an efficiency of transport index which was calculated by dividing the concentration of the sample by the discharge at the time of sampling (Figure 5). From Figure 5 it can be seen that the high indices of efficiency of transport occur when more than 70% of the particles are smaller than medium silt size. However, some low efficiency of transport indices are encountered (ringed in Figure 5) when the silt is fine and it is likely that these discrepancies occur when a period of low availability coincides with a period of moderate transport potential, for example, in the inter-peak period of the efficiency of transport curve (Figure 4) prior to the increase of erosive scour associated with high transport potential. The curved line in Figure 5 has been drawn in by eye to illustrate the interpreted relationship and has no statistical application. It is considered that the quality of the data is such that a statistically derived best fit curve would have little meaning, but the data has served to illustrate that relatively fine sediment is generally associated with high transport efficiency. It is anticipated that similar results would have been obtained if a different standard silt size had been chosen from the medium silt class.

Summary of factors effecting the seasonal distribution of sediment load.

During the dry period transport potential limits the removal of sediment and is less than the rate of production with the result that sediment accumulates on the catchment surface. With the first sharp increase in runoff at the beginning of the wet period the fine accumulated sediment is carried away so that the catchment is 'flushed' and any further sediment produced is removed at the optimum rate. The amount of sediment reaching the streams is now limited by the rate of production which increases with the increased rate of chemical weathering and the erosive scour associated with the higher flows. The rate of production decreases with decreasing runoff at the end of the wet season until there is no longer sufficient overland flow to remove all the sediment being produced and the cycle starts again.

The general principles that have been used to explain the 'normal' (double peak) pattern of variation of sediment load will also be applied to the exceptional case of the triple peak pattern observed at Station D7M05 at Upington on the Orange River (Figure 2). The Orange River at Upington is fed by three main contributing catchments, namely, the Orange catchment above the Vaal confluence, the Vaal catchment above the Hartz confluence and the Hartz catchment. An examination of the suspended sediment records at Stations D1M03 (Orange River) and C3M03 (Hartz River) as well as an examination of suspended sediment data for the Vaal River published by McCrae (1945) shows that all three catchments are characterised by the double peak pattern in the seasonal variation of sediment load. The contribution of sediment to Station D7M05 from three large catchments leads to the supposition that the triple peak has been caused by a difference in the seasonal rainfall pattern between the three catchments with the result that the double peak patterns for the three catchments are not in phase. However, the single peak in the flow curve for Station D7M05 (Figure 2) indicates that this is not the case as a difference in the flow regime for any one of the catchments would be evident in the regime of the main stream. The peak discharge coincides with the second peak in the sediment load (Figure 2) and consequently the third and highest peak is associated with decreasing runoff at the end of the wet season. In terms of the variation of catchment conditions, this situation could only arise if the rate of production did not decrease concordantly with transport potential at the end of the wet season, but increased again for a sufficient period of time to give rise to a third peak in the removed sediment curve. Bearing in mind that the main contributing catchments to Station D7M05 conform to the 'normal' double peak pattern in the variation of sediment load, the reasons for a short term increase in the rate of production during a period of decreasing transport potential should be sought in the lower Orange River catchment. The research would involve a detailed analysis of the catchment conditions, the pattern of discharge and the pattern of water usage in the lower Orange River. As the aim of this study is to explain the general pattern of variation the exception of Station D7M05 will not be investigated further in this study.

It has been reasoned that the transport potential and the rate of production of sediment are the two factors that limit the quantity of sediment reaching the stream channels, and are the two main factors that control the seasonal variation of sediment yield. Both the transport potential and the rate of production, as defined, are themselves known to be controlled by a large number of interrelated variables which can be broadly classified into three groups; climate, vegetation and soil variables. "Factors of rainfall, soil character and vegetation are dependent on climate and geology and are interrelated to each other. All these not only influence the amount and rates of erosion but also the size and shape of the landscape created by the streams" (Morisawa, 1968, p 64). If the seasonal interaction of climate, vegetation and soil variables give rise to the transport potential and rate of production characteristics that control seasonal sediment yield, it is reasonable to assume that the long term interrelationship between the three groups of variables controls the long term variation in sediment yield, that is, the mean annual sediment yield.

The use of transport potential and rate of production as prediction variables in a model would involve the measurement of all the variables affecting them as well as a knowledge of the complex interrelationships between the variables. Such an approach is not feasible and it would be more profitable to adopt an indirect approach by choosing an easily measured prediction variable that is also controlled, to a greater or lesser degree, by the complex interrelationship between climate, vegetation and soil variables. The long term interrelationship between these three groups of variables forms the basis of the concept of a drainage basin as an open system in a steady state. In the following chapter it will be shown that the concept of steady state can be used to relate mean annual sediment yield to the morphology of the channel network.

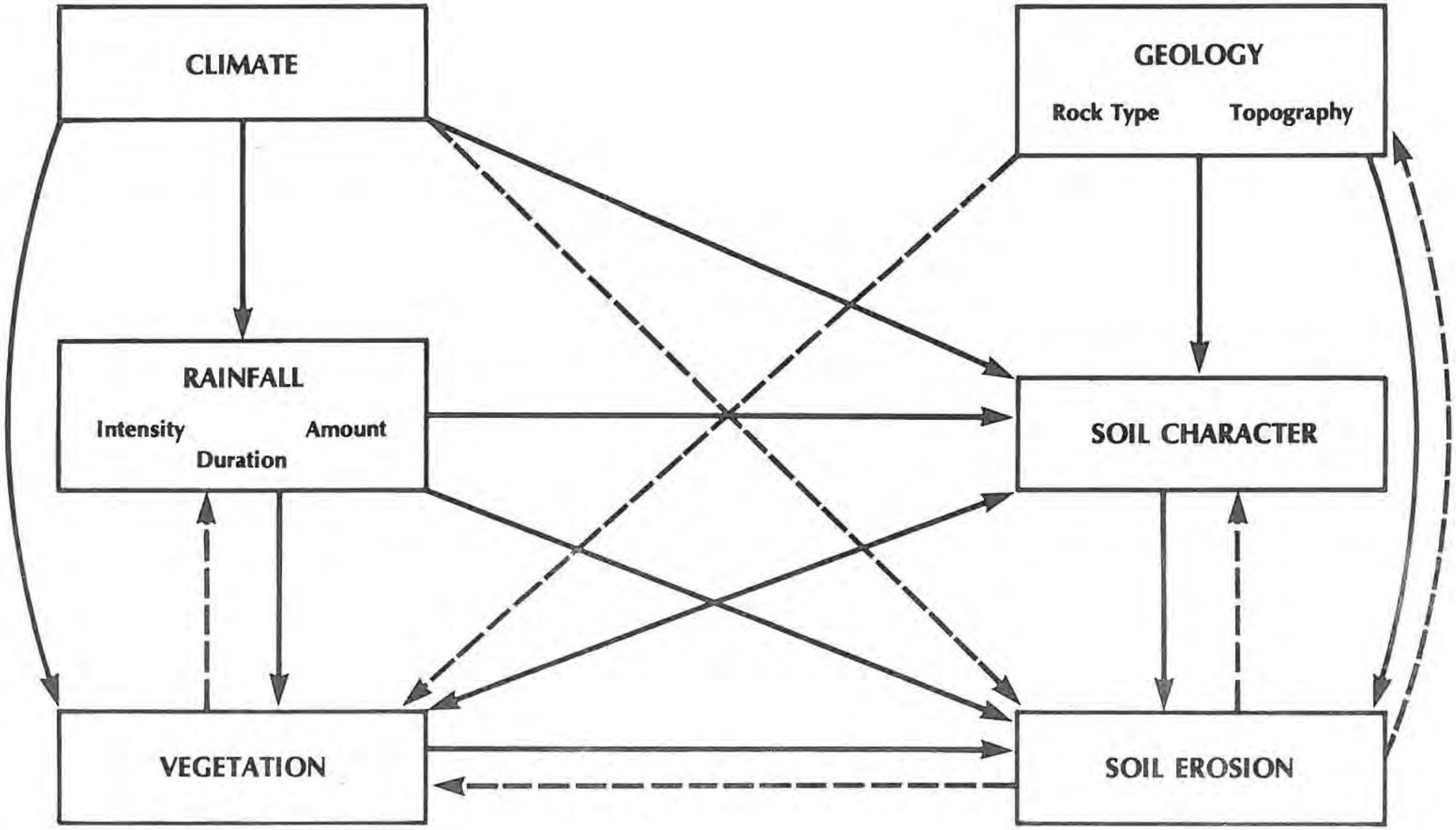
CHAPTER IV

THE CONCEPT OF A DRAINAGE BASIN AS AN OPEN SYSTEM IN STEADY STATE

A drainage basin can be considered as an open system because energy and matter both enter and leave the system. "In open systems the rates of import and export of material and energy will become balanced so that an equilibrium called the steady state is reached. The system is self regulating; any change in the controlling factors will cause a shift in the steady state that will tend to absorb the effects of the change" (Morisawa, 1968, p 126). The concept of steady state in a catchment is based on the interrelationship of climate, vegetation and soil variables. The climate variables form the input to the open system and interact with the vegetation and soil variables of the catchment surface under steady state conditions to define the output in the form of sediment load and runoff. Over a period of time the input and output of material and energy are balanced so that the output is a function of the long term interrelationship between vegetation, soil and climate variables, with the climate variables being considered as essentially independent. The steady state conditions control the morphology of the landscape and in this respect Tricart and Cailleux (1972, p 164) have referred to the steady state as a morphoclimatic equilibrium defined as a "morphoclimatic adjustment that is realised in a given region when the land forms are predominantly determined by a morphogenic system that is dependent on climatic factors". The use of the word equilibrium when referring to steady state conditions can be misleading because of the early concepts of equilibrium in which an equilibrium was a limit that was approached but could never be attained. Baulig, for example, gave the following definition of the concept of equilibrium applied to profiles, "An actual profile of equilibrium is always provisional; it is never definite or final. All that can be said is that, given an evolution free of interruptions, the profile proceeds increasingly slowly towards a limit which, by definition it cannot attain" (Tricart and Cailleux, 1972, p 161). This definition contrasts with later definitions in which the profile of equilibrium is reached when the gradient has been reduced to such an extent that there is no further change. Both concepts of equilibrium do not conform with the concept of steady state since in the first case steady state is achieved and maintained and secondly, the concept of stability rarely exists in nature. Careful observations of stream courses have shown that minor or major alterations take place continuously with the result that geomorphologists have tended to refer to quasi or dynamic equilibrium which is more in accordance with the concept of steady state. Tricart and Cailleux (1972, p 160) explain their use of the word equilibrium in reference to steady state as follows, "The morphoclimatic system of any climate zone is dependent not only on climate but on vegetation and soil. It is on the interrelationship of these three factors that the concept of morphoclimatic equilibrium is based. The concept of morphoclimatic equilibrium, actually, is less a balance than a permanent state or steady state in the sense of the physicist. The fact that stream water always holds more foreign matter in suspension or solution than rainwater demonstrates that there is always erosion and that watercourses and slopes shift even if their profiles remain constant. The permanence of forms during a given period is, however, so frequent and so important that we conform to usage and refer to it in terms of an equilibrium".

Steady state and the mean annual sediment yield

Mutual interaction of the climate, vegetation and soil variables of a watershed system tend towards the establishment and maintenance of a steady state. The interaction of the three groups of variables also control the amount and rate of erosion as illustrated in the flow chart of factors which influence soil erosion shown in Figure 6. It is therefore reasonable to



After Morisawa (1968, p 78)

Figure 6 Flow Chart of Factors which Influence Soil Erosion assume that the mean annual sediment yield is characteristic of the steady state conditions and that a shift in the steady state will give rise to a corresponding change in the mean annual sediment yield. For example, an upward shift in the steady state due to decreased resistance of the catchment (caused by long term deterioration in soil or vegetation conditions) to climate input would cause an increase in mean annual sediment yield. Support has been given to this assumption by the illustration in Chapter III that even short term (seasonal) variations in sediment yield can be ascribed to the interrelationship of climate, vegetation, and soil variables.

Steady state and channel morphology

The output from the open watershed system takes the form of sediment load and runoff which are discharged from the system by the channel network. Consequently the channel network is also characteristic of the steady state[9] and can be thought of as the physical expression of the output from the system. Any environmental change which upsets the steady state will be compensated for in the morphology of the channel network in order to maintain the balance between its ability to transport and the load provided.[10] The river will adjust to the new conditions by changing its slope, cross-section, roughness of bed, length of channel, channel pattern or a combination of these factors. Under steady state conditions the morphology of the river is not static because "within the same drainage basin tributaries are being added in some places and lost in others, some streams are being lengthened and some streams are being shortened. The adjustments in the watershed system are tending towards the establishment and maintenance of a steady state" (Morisawa, 1968, p 164). Thus, although the steady state is independent of time, the morphology of the channel network is not static and achieves and maintains steady state conditions by mutual interaction of channel characteristics.

The concept that a channel network achieves steady state conditions by mutual interaction of channel characteristics is important as it infers that even non-graded streams may be in steady state by adjustment of cross-sectional form and roughness. Leopold and Miller (1956, p 1) found in their study of the hydraulic geometry of river channels that, "The tendency for stream channels to maintain a quasi-equilibrium with imposed discharge and load is shown to be characteristic of ephemeral channels in the headwaters of the drainage basin, even to the most headward rill". The term steady state can therefore be applied to rivers which have reached a stable condition of self-regulation even though the profile is constantly changing.

Since the mean annual sediment yield of a catchment and the channel morphology are both characteristic of, and sensitive to, changes in the steady state condition there should be a relationship between channel morphology and mean annual sediment yield. In order to relate channel morphology to mean annual sediment yield it is necessary to choose some morphological index that is representative of the entire network and one which is likely to be sensitive to changes in steady state conditions. The index chosen for this purpose is the channel storage capacity per unit of drainage area as measured by Horton's P ratio (Horton, 1945). Storage capacity in this context refers to the volume occupied by the channels and is not only representative of the average load provided, but will also vary with changes of load. The P ratio is obtained from the map plan of the channel network which is an additional advantage because, "Investigations show that not only are cross-sectional morphology and longitudinal profile of a river adjusted to environmental controls of load and discharge, but also is the map plan of the channel network" (Morisawa, 1968, p 135). If the foregoing assumptions are true both the mean annual sediment yield and the P ratio are controlled by the same set of environmental conditions that give rise to steady state and should therefore vary in parallel with one another as steady state conditions vary from one catchment to another.

The model for the estimation of mean annual sediment yield in ungauged catchments can therefore be based on the following hypothesis:

At any chosen point in a river system there is a relationship between the mean annual sediment yield per unit of drainage area and the P ratio of the channel network above that point, provided the watershed system conforms to the following conditions:

(i) steady state conditions prevail in the drainage basin, and

(ii) Horton's two laws of stream lengths and stream numbers hold true for the drainage network.

The P ratio is the ratio of the length ratio (law of stream lengths) to the bifurcation ratio (law of stream numbers) for any stream network. As only ratios are involved in the measurement of the P ratio, values can be readily obtained for any catchment in South Africa by using topographical maps at any convenient scale. Before testing the hypothesis some aspects of morphology related to the derivation and interpretation of the P ratio will be examined.

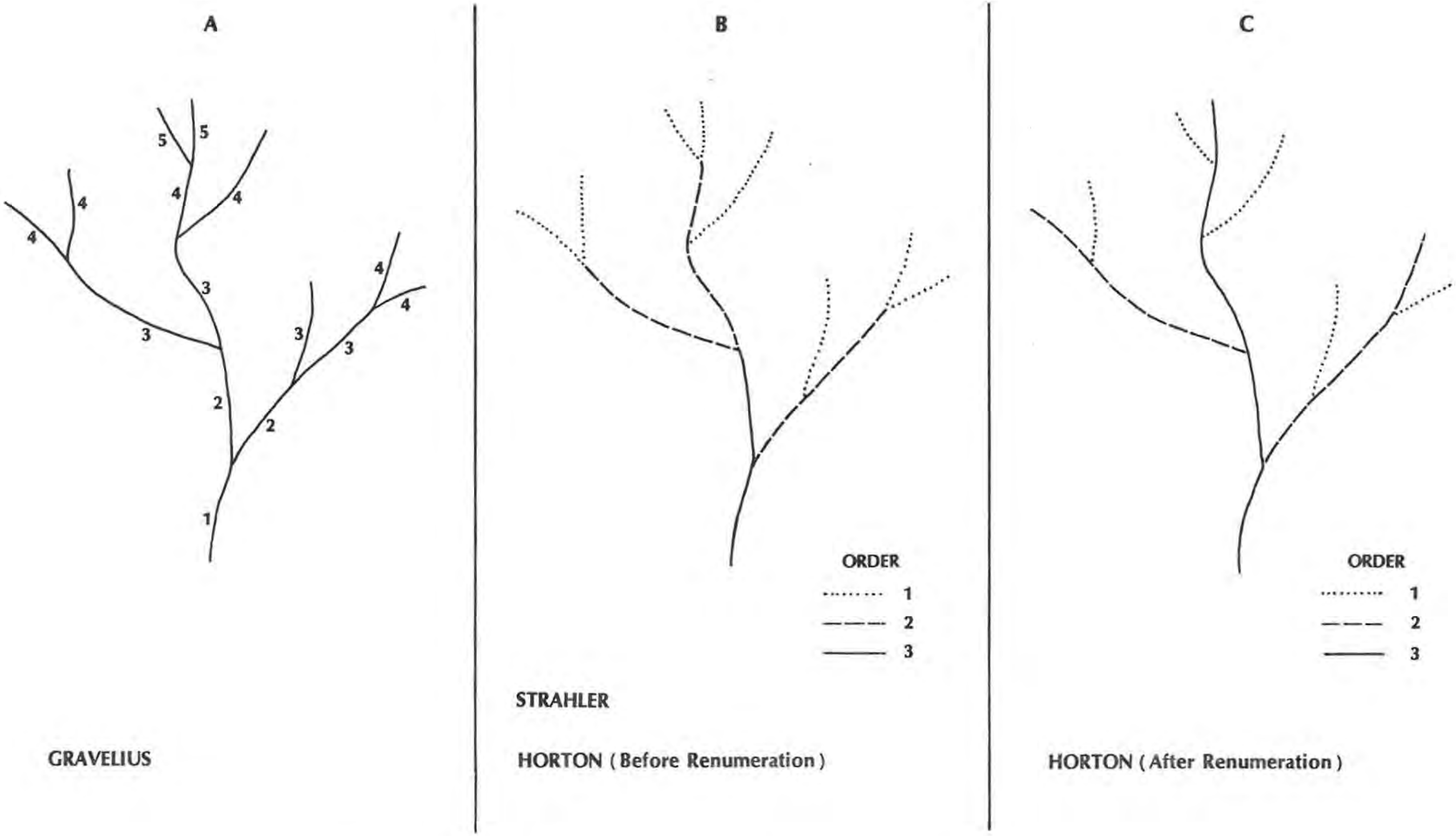
CHAPTER V

THE DERIVATION OF HORTON'S P RATIO

The derivation of Horton's P ratio (Horton, 1945) for any channel network necessitates a basic working knowledge of those aspects of channel network morphometry related to the systematic ordering and segmentation of stream channels. More specifically, the P ratio is calculated from the bifurcation ratio and length ratio for a network which involves the ordering of the channels and the measurement of their length. Horton's method of stream ordering will be discussed and briefly compared with a method devised by Strahler (1964). Strahler's method will not be used for the derivation of the P ratio, but it is of relevance to the application of network topology discussed in Chapter VI.

One of the most important aspects of any quantitative analysis of a drainage basin is the concept of stream ordering which was introduced by Gravelius in 1914 (Figure 7a). The work of R.E. Horton (1945) marked the beginning of a widespread use of channel ordering in morphometric analysis, and he suggested a method of classifying streams and drainage basins which has become known as the 'Horton analysis of drainage composition'. The composition of a drainage basin refers to the number and length of streams of different orders regardless of the pattern (map plan) of the channels in the drainage basin. In terms of a hydrological approach to the analysis of drainage basins, Horton considered the composition to be more important than the pattern of the stream network because two stream networks that conform to the same pattern (e.g. trellis or dendritic) could have markedly different compositions.

Horton ordered the hierarchy of tributaries by calling the fingertip or unbranched tributaries first order streams. The confluence of two first order streams gave rise to a second order stream and only when two second



GRAVELIUS

STRAHLER

HORTON (Before Renumeration)

HORTON (After Renumeration)

Figure 7 An Illustration of Methods of Stream Ordering order streams met was a third order stream formed. In this way streams were designated higher and higher orders up to the main stream which was of the highest order and determined the order of the drainage basin. To illustrate the hierarchy of stream orders Horton's method of stream ordering has been applied to a hypothetical drainage network in Figure 7b. Horton's method of ordering streams is the inverse of the European system first used by Gravelius. In the European system the main or stem stream is designated an order of one and each upstream junction gives rise to two streams of the next higher order with the result that the unbranched fingertip streams have higher orders than the main stream (Figure 7a). Horton's method is more logical because the unbranched fingertip streams, which are similar in character, are all given the same order and the main stream has the highest order giving an indication of the magnitude of the network.

Once the hierarchy of streams had been ordered, Horton measured the length of each order stream to determine the average stream length of each of the different orders. The length of a stream of any order was measured from its confluence with the next higher order stream (or the sea in the case of the main stream) to the extremity of its longest, most headward, first order tributary. However the longest, most headward tributary (the source of the river) had been designated an order of one and as it was part of a higher order stream it was renumbered accordingly. For example, those first, second, third and fourth order streams that form the headward segments of a fifth order stream were all renumbered to fifth order (Figure 7c). If a stream system has a high density of channels, it is sometimes difficult to choose for renumeration those headward segments that will give the main stream its greatest headward length. In order to determine which tributary was a parent stream Horton started downstream of any junction and extended the parent stream headwards along that tributary which joined the parent stream at the smallest angle. In cases where both tributary streams were at the same angle to the parent stream at the junction, the parent stream was extended along the longest tributary.

In order to avoid the necessity of subjective decisions during renumeration Strahler (1964) modified Horton's method of ordering by restricting the designation of order to stream segments (Figure 7b). The restriction of stream order to stream segments obviates the need for renumeration as the length of each segment extends headwards only as far as the confluence of two streams of the next lower order. The modification simplifies computation but progressively shortens the length of the higher order channels because their headward components under the Horton method have not been included in their length. However, stream lengths measured by following the two methods may still be equated if Broscoe's length correction (Broscoe, 1959) is applied when using Strahler's method. Broscoe plotted the log of the mean cumulative length of streams in each order basin against stream order to include the length of the headwater tributaries in the stream length of the higher order basins.

Both Horton's and Strahler's methods of stream ordering have an inherent disadvantage in that the distribution of orders is not always consistent with stream bifurcation. The inconsistency arises from the fact that the entry of a lower order tributary does not always increase the order of the main stream. Stream networks are often more dense on slopes of exposed, well jointed bedrock than they are on well vegetated slopes. The bedrock slopes tend to introduce additional first and second order streams which would have a direct consequence on the ordering of the rest of the system only if the additional streams flowed into streams of the same order. The addition of first and second order streams to a third order stream would have no effect on the ordering of the network downstream. Strahler's method has the disadvantage in that the junction of two streams just before a confluence with a higher order channel can give rise to a decrease of stream length with increasing order. Despite the disadvantages associated with the Horton and Strahler methods of stream ordering, they have become firmly established in morphometric analysis.

As a result of the application of stream ordering to a large number of drainage basins, Horton found a number of relationships which he stated in the form of five laws of drainage composition. Only the first two laws are of relevance to the derivation of the P ratio and they are the law of stream numbers and the law of stream lengths.[11]

The law of stream numbers

The first law of drainage composition was the law of stream numbers which stated that, "The number of streams of different orders in a drainage basin tend closely to approximate an inverse geometric series in which the first term is unity and the ratio is the bifurcation ratio " (Horton, 1945, p 291). The bifurcation ratio (rb) was introduced by Horton to express the ratio of the number of streams of any given order to the number in the next higher order, and gives a measure of the degree of branching of streams in the river system.

The law of stream lengths

The second law was the law of stream lengths which stated that, "The average lengths of streams of each of the different orders in a drainage basin tend closely to approximate a direct geometric series in which the first term is the average length of the first order" (Horton, 1945, p 291). The length ratio (rl) for the system is the ratio of the average length of any given order to the average length in the next lower order. In both laws the geometric series is approximated with the result that there is usually a slight variation in the bifurcation or length ratio between successive orders. Consequently the bifurcation and length ratios for the network as a whole are best calculated by taking the average of the ratios between successive orders.

For a given density of drainage the bifurcation and length ratios are markedly influenced by basin shape. In homogeneous bedrock the bifurcation ratio and length ratios show little variation from one area to another, but where structural control causes basin elongation the values of the two ratios may increase appreciably. The degree of stream branching as given by the bifurcation ratio has an important control over the peakedness of the runoff hydrograph because low bifurcation ratios result in fewer but larger contributions to the runoff in the main channel.

Horton hypothesised that the P ratio, which is the length ratio (rl) over the bifurcation ratio (rb), would give the degree of drainage development because it was a measure of channel storage per unit of drainage area. Channel storage in this context refers to the total volume occupied by all the channels of the network. Different stream systems may have approximately the same drainage density and yet differ markedly in channel storage capacity. The higher order channels have larger cross-sections and contain more channel storage per unit length than lower order channels. Therefore, if the P ratio is high, the greater length of larger stream channels would afford greatly increased channel storage per unit of drainage area. The interpretation of the P ratio as a relative indicator of total channel volume per unit of drainage area makes it one of the few network indices that is representative of the morphology of the entire channel network. Because the P ratio can also be calculated for the drainage network above any point of reference in the system it is possible to examine the relationship between the P ratio and the mean annual sediment yield at any point in a river system. The fifteen reservoirs listed in Table 1 will be used as reference points and the P ratios of the channel networks feeding the reservoirs will be calculated in Chapter VI to test the hypothesised relationship between mean annual sediment yield and the chosen variable.

CHAPTER VI

THE FORMULATION OF THE MODEL

It was hypothesised in Chapter IV that at any chosen point in a river system there is a relationship between the mean annual sediment yield per unit of drainage area and the P ratio of the channel network above that point, provided the watershed system conforms to the following conditions;

(i) steady state conditions prevail in the drainage basin, and

(ii) Horton's two laws of stream lengths and stream numbers hold true for the drainage network.

The hypothesis will be tested by examining the relationship between the mean annual sediment yields measured at the fifteen reservoirs listed in Table 1 and the P ratios of the drainage networks feeding the reservoirs. A relationship between mean annual sediment yield and Horton's P ratio will be used to formulate a model for the estimation of mean annual sediment yield in ungauged catchments and deviations from the relationship will be explained in terms of non-conformity with one or more of the conditions laid down in the hypothesis.

Methodology

One of the principal advantages of choosing Horton's P ratio as a prediction variable is that it can be calculated from the map plan of the drainage network. In large catchments where the drainage network covers an area of 10 000 square kilometres or more, the ordering and measurement of every channel becomes a tedious and time consuming process. Fortunately the nature of the P ratio is such that it is not necessary to use a map of sufficiently large scale to show every channel. According to the laws of stream numbers and stream lengths, the bifurcation and length ratios for any channel network remain constant from the lowest to the highest order streams and therefore any suitable map scale may be chosen for the purpose of obtaining a value for the P ratio.

The drainage systems upstream of the fifteen reservoirs listed in Table 1 were traced off topographical maps at a scale selected for each system so that the system was represented at approximately the fifth order. The fifth order systems were found to be of a managable size and represented a sufficient number of channels to accurately determine the bifurcation and length ratios. The channel networks were ordered according to Horton's method of ordering and the bifurcation ratios were calculated prior to the remuneration[12] of stream segments. If the bifurcation ratio is calculated after remuneration[13] it is not as representative a measure of stream branching because there is a marked increase in the number of stream entries that do not result in an increase of stream order. Once the bifurcation ratios had been calculated the parent streams were renumbered and the lengths of the channels were measured. The length ratio for each network was then calculated from the average lengths of each of the different order channels. The P ratios were calculated[14] by dividing the length ratio by the bifurcation ratio for each network (Table 1).

The mean annual sediment yield for each system was calculated from the trigonometric surveys for each reservoir and expressed as a volume (of wet sediment) per square kilometre of catchment area (Table 1). In the case of the smaller reservoirs that have a full supply capacity that is less than the mean annual runoff of the river network feeding them, some adjustment to the mean annual sediment yield was necessary to allow for lower trap efficiency.[15] For this purpose a trap efficiency correction index, $T = \frac{100 - C}{10}$, was devised where T is the percentage of the mean annual sediment yield to be added and C is the full supply capacity of the reservoir expressed as a percentage of mean annual runoff (Table 1).

A plot of the P ratios against mean annual sediment yield in Figure 8 shows that thirteen of the fifteen points lie very close to the straight line while the remaining two points (marked with crosses in Figure 8) deviate markedly from the general trend line. The two points that deviate from the trend line are the values for Van Ryneveld's Pass Dam and Lake Mentz, both on the Sundays River. A linear least squares correlation analysis gave a correlation coefficient of 0,87 for the relationship, but if the Sundays River is excluded and a correlation analysis conducted for the remaining thirteen systems, the correlation coefficient rises to 0,99 significant at the 0,95 level. The influence of the trap efficiency correction index on the relationship was tested by calculating the correlation coefficient between the P ratios and the mean annual sediment yields without adjustment for trap efficiency. The correlation coefficient was found to be 0,98 showing that the trap efficiency index has provided a slight improvement by raising the correlation coefficient to 0,99. The sharp rise in the correlation coefficient on the exclusion of the Sundays River system illustrates that the Sundays River system deviates markedly from a relationship that exists between the P ratio and the mean annual sediment yield in all the other catchments. In view of the high significant correlation that has been found between the P ratio and the mean annual sediment yield per unit area, the hypothesis can be accepted as valid. The deviation of the Sundays River from the relationship can be attributed to non-conformity with one, or both of the conditions laid down in the hypothesis. The non-conformity of the Sundays River system will be examined in some detail before proceeding with the formulation of a model based on the relationship found in the other thirteen systems.

The calculation of the bifurcation and length ratios for both the Van Ryneveld's Pass Dam and Lake Mentz networks showed that the Sundays River system conforms to Horton's laws of stream lengths and stream numbers. As one condition of the hypothesis has been complied with, the possibility that there is non-conformity with the second condition, namely that of steady state, should be examined. Unfortunately there is no ready method of testing whether or not a system is in steady state and graded profiles cannot be used as indicative because the concept of steady state can be applied even to non-graded streams. However, if the steady state has been upset by environmental changes causing an increase in sediment load (the Van Ryneveld's Pass Dam and Lake Mentz networks carry more sediment than their P ratios would suggest as shown in Figure 8) it can be assumed that the morphology of the channel network is also adjusting to accomodate the greater load. If the characteristic bifurcation ratio remains constant during network growth the exterior low order streams (growth elements) of the network must bifurcate more rapidly than the more stable interior high order streams and consequently the response of a system to an upward shift in steady state would be most marked in the exterior growth elements of the network. By using a topological measure of network growth process, the E index (Jarvis, 1972), it is possible to test whether the ratio of the exterior growth elements to the more stable interior elements is greater for the Sundays River system than for the other thirteen systems. If the growth elements of the Sundays River system are found to be more 'dynamic' with respect to internal elements as compared with the other systems, it may be interpreted as indicative of a response to an upward shift in steady state. The hypothesis that the ratio of exterior growth elements to the more stable interior elements is greater for the Sundays River system than for the other thirteen systems examined, will be tested by deriving an E index for each system.

The E index

The E index is a measure of the topological structure of dendritic drainage networks and Jarvis (1972) has shown that it can be used as an aid to the interpretation of network growth processes. The index has not, as yet, been extensively applied but the results obtained by Jarvis indicate that it will prove to be a valuable tool in the field of network topology. In order to facilitate the explanation of the topological terminology used by Jarvis in the derivation of the E index, a hypothetical drainage network has been drawn in Figure 9a together with a topological representation of the same network (Figure 9b). The topological representation consists of a system of nodes (source or junction nodes) connected by straight line linkages, that is, the channel segments between stream junctions in the 'normal system' lose their characteristic length and azimuth to become simple linkages between nodes. The topological linkages may be ordered in the same way as any stream network using any particular ordering system. However, the use of the Horton or Strahler methods of stream ordering is not recommended by Jarvis for the derivation of the E index because both methods give rise to deviation from the distribution law, that is, stream entry does not always increase the order of the parent stream. To meet the requirements of the distribution law Shreve (1966) derived a system based on stream magnitude rather than stream order and the Shreve system was chosen by Jarvis for the derivation of the E index.

When applying the Shreve system to a topological representation of a drainage network, all exterior linkages are given a magnitude of one and at any interior junction node the magnitude of the downstream linkage is determined by adding the magnitudes of the two upstream linkages. The magnitudes of the linkages are then transferred to the nodes so that any node has the magnitude of the downstream link emanating from it (Figure 9b). When the nodes have been given magnitude their positions in the network with respect to the root (mouth) of the network are measured in terms of link distance. The link distance of any node is defined as the number of linkages from the root to the node by the most direct flow route through the network. For example, the junction node marked A in Figure 9b has a link distance of seven and a magnitude of two.

Jarvis used the link and magnitude concepts to develop a more sophisticated topological measure of network structure which he called the E index. The E index is given by;

$$E = \frac{\Sigma MHi}{\Sigma MHe} \qquad \text{(Eqn 2)}$$

where M is the magnitude of a given node and H is its link distance. The subscript i and e denote summation over the interior nodes (junction nodes) and over the exterior nodes (source nodes) respectively. The magnitude parameter summarizes the amount of drainage development headward of a given node and the link distance parameter summarizes the structural configuration of the network downstream of the node. The ratio expressed by the E index combines these properties in terms of the relationship between the more stable internal elements of the network and the most active (exterior) growth elements.

Jarvis demonstrated that the E index tends to increase with Shreve's system magnitude and observed that the rate of increase of the E index with magnitude varies from one Strahler order system to another (Jarvis, 1972, p 1270, Figure 3). The fifteen river systems to be analysed in this study comprise a wide range of both system magnitude and Strahler order, and as a result the influence of both system magnitude and Strahler order on the E indices must be removed before they can be compared with one another.

The morphometric analysis of the fifteen river systems listed in Table 1 has shown that all the river systems conform to Horton's law of stream numbers. As the bifurcation ratio remains constant in each river system and because the E index is also an intra-system ratio, the river systems may be analysed at any convenient scale. Topological representations of the fifteen river systems were drawn and the nodes were ascribed magnitude and link distance according to the Shreve system. An E index for each river system was then calculated by using Equation 2 and plotted against the respective system magnitude in Figure 10. The values of E index and Shreve's system magnitude are listed in Table 3. The plot of E index against system magnitude (Figure 10) demonstrates that the E index increases with system magnitude at a different rate for each Strahler order system. A regression of E index on system magnitude for all the systems gives a common regression line marked

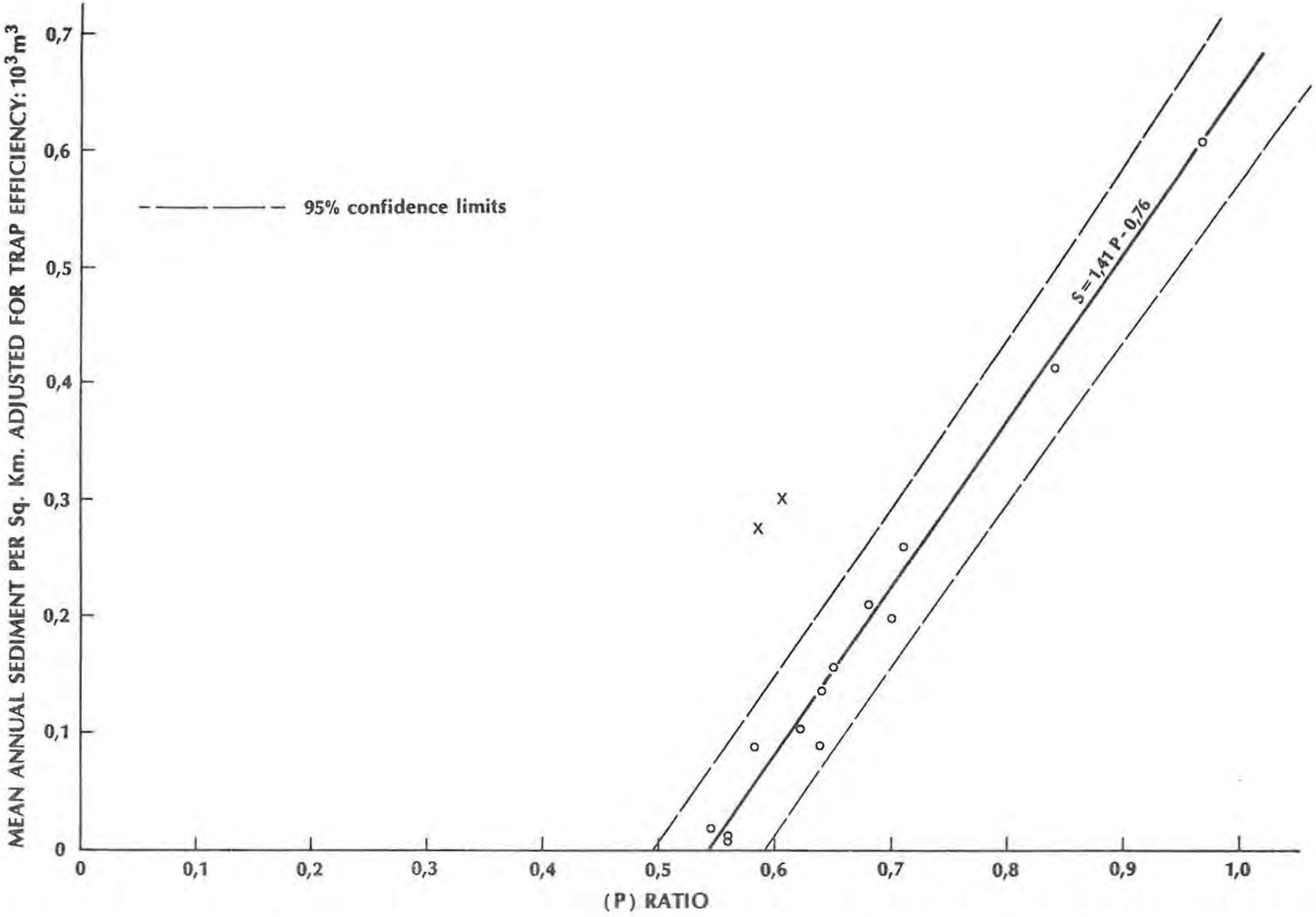
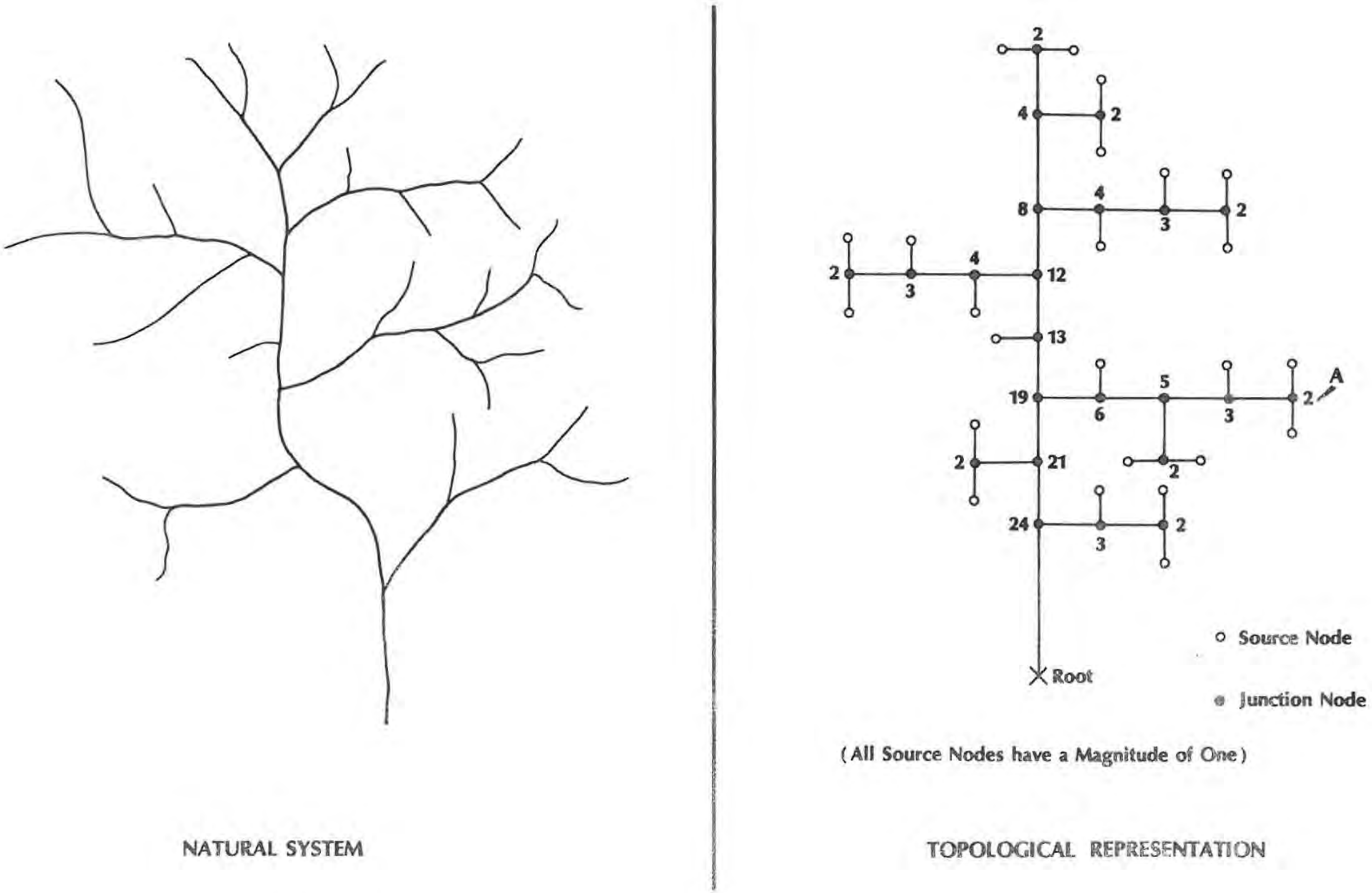


Figure 8 The Relationship between Mean Annual Sediment Yield and Horton's P Ratio



NATURAL SYSTEM

TOPOLOGICAL REPRESENTATION

Figure 9 A Topological Representation of a Drainage Network

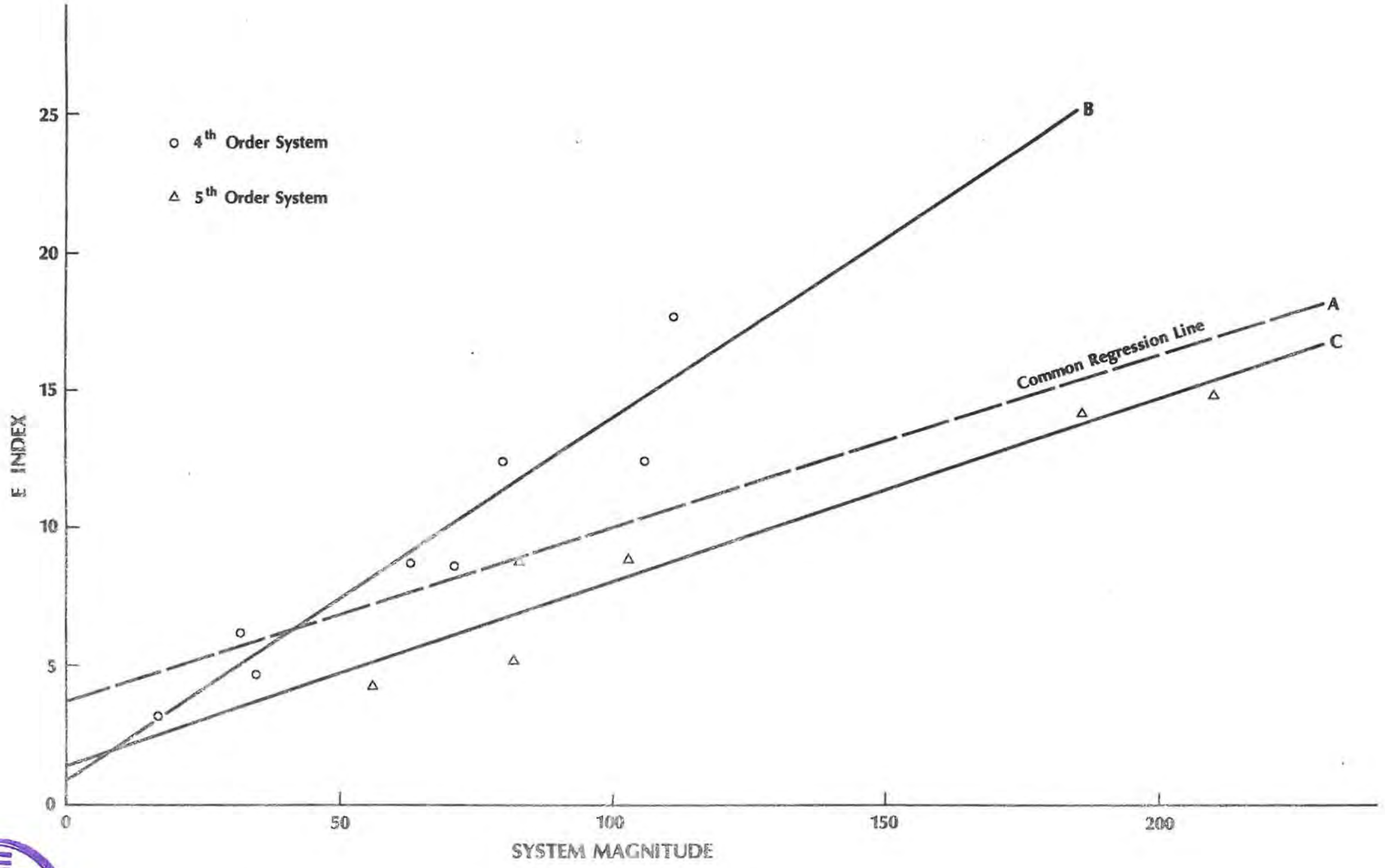


Figure 10 The Variation of the E Index with Shreve's System Magnitude

TABLE 3

E INDICES AND SYSTEM MAGNITUDES

Reservoir System	Code Number	E Index	Strahler Order	Shreve System Magnitude
Grass Ridge	Q1R01	5,19	5	82
Oukloof	J2R03	12,39	4	106
Schweitze-Reneke	C3R01	8,64	4	71
Olifants Nek	A2R03	4,32	5	56
Koppies	C7R01	14,85	5	210
Albasini	A9R01	4,65	4	35
Tierpoort	C5R01	8,75	4	63
Van Ryneveld's Pass	N1R01	3,18	4	17
Lake Arthur	Q4R01	14,13	5	186
Lake Mentz	N2R01	12,40	4	80
Egmont	D2R01	6,19	4	32
Kommando Drift	Q4R02	8,89	5	103
Kammanassie	J3R01	17,74	4	111
Prinsrivier	J1R01	8,81	5	83
Loskop	B3R02	19,35	6	427

TABLE 4

THE TRANSFORMED, EXPECTED AND STANDARDISED E INDICES

Reservoir System	E(T) Transformed	E(E) Expected	E(S) Standardised	E(S+5)
Grass Ridge	7,37	8,90	-1,53	3,47
Oukloof	9,29	10,41	-1,12	3,88
Schweitze Reneke	7,49	8,21	-0,72	4,28
Olifants Nek	6,55	7,26	-0,71	4,29
Koppies	16,45	16,97	-0,52	4,48
Albasini	5,57	5,94	-0,37	4,63
Tierpoort	7,54	7,71	-0,17	4,83
Van Ryneveld's Pass	4,87	4,81	0,06	5,06
Lake Arthur	15,77	15,45	0,32	5,32
Lake Mentz	9,29	8,78	0,51	5,51
Egmont	6,31	5,75	0,56	5,56
Kommando Drift	10,85	10,22	0,63	5,63
Kammanassie	11,86	10,73	1,13	6,13
Prinsrivier	10,77	8,96	1,81	6,81
Loskop	-	-	-	-

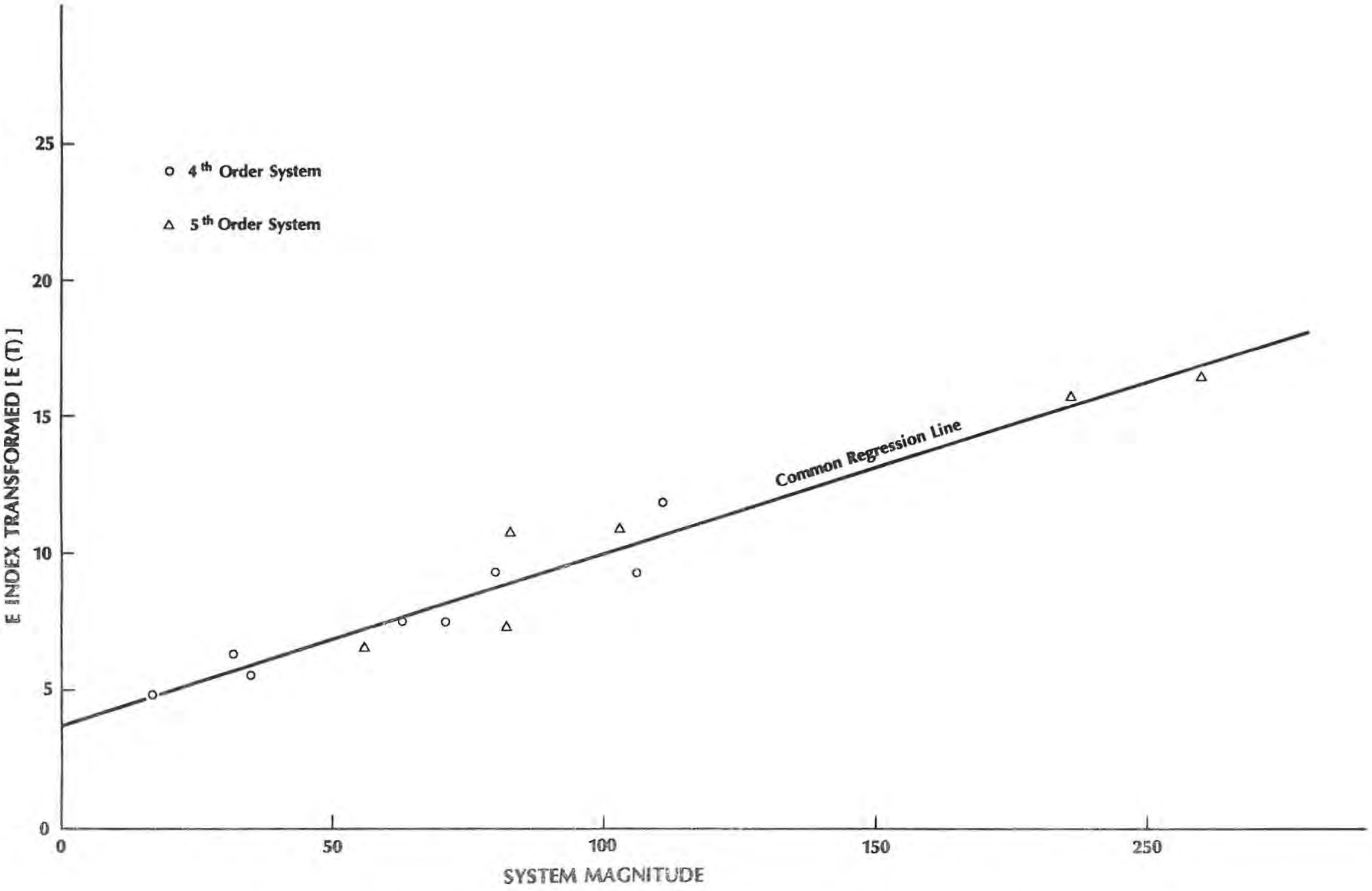


Figure 11 The Variation of the Transformed E Index with Shreve's System Magnitude

(A) in Figure 10 with a correlation coefficient of 0,77. However, if the relationship between E index and system magnitude is investigated within each Strahler order, the correlation coefficient rises to 0,95 for both Strahler fourth and fifth order systems. Separate regression lines were calculated for Strahler fourth and fifth order systems and have been drawn in Figure 10 as lines (B) and (C) respectively. No regression line could be obtained for sixth order systems as the Loskop system is the only system of the sixth order.

It is necessary to remove the influence of system magnitude from the E indices before the values of the E indices can be compared, but as the relationship between magnitude and E index is not the same for all systems, the effect of Strahler order on the relationship must be removed first. The effect of Strahler order may be removed by transforming the relationship between E index and system magnitude in each order system to some arbitarily chosen common rate of increase of E index with magnitude.[16] The common regression line (A) in Figure 10 provides a convenient standard for the transformation, and the E indices were transformed to this regression line by using a technique that preserved the characteristic scatter of the points about the regression lines for each Strahler order. The technique developed by the author is illustrated below:

The regression equation for common regression line (A)[17] is

$$E_c = 0{,}063M + 3{,}736 \qquad \text{(Eqn 3)}$$

and the regression equation for fourth order systems (B) is

$$E_4 = 0{,}131M + 0{,}809. \qquad \text{(Eqn 4)}$$

The regression line for fourth order systems can be transformed to the common line by multiplying the fourth order E index values by 0,48 and adding 3,348 to them to give;

$$(E_4 \times 0{,}48) + 3{,}348 = 0{,}063M + 3{,}736 \qquad \text{(Eqn 5)}$$

$$\therefore (E_4 \times 0{,}48) + 3{,}348 = E_c. \qquad \text{(Eqn 6)}$$

Similarly for the fifth order systems the regression line (C) is

$$E_5 = 0{,}067M + 1{,}316 \qquad \text{(Eqn 7)}$$

and can be transformed to the common regression line (A) by multiplying the fifth order E index values by 0,94 and adding 2,499 to them to give;

$$(E_5 \times 0{,}94) + 2{,}499 = 0{,}063M + 3{,}736 \qquad \text{(Eqn 8)}$$

$$\therefore\ (E_5 \times 0{,}94) + 2{,}499 = E_c. \qquad \text{(Eqn 9)}$$

The transformed values of the E index (E(T)) are listed in Table 4 and have been plotted against their respective system magnitudes in Figure 11 to show the result of the transformation. As there is a common rate of increase of transformed E index (E(T)) with system magnitude, the influence of magnitude may be removed by calculating the expected E index (E(E)) for each system magnitude according to Equation 3 and subtracting this expected E index (E(E)) from the transformed E index (E(T)) to obtain a standardised E index (E(S)) expressed as

$$E(S) = E(T) - E(E). \qquad \text{(Eqn 10)}$$

The standardised E indices are listed in Table 4. Some of the transformed E indices are smaller than their associated expected E indices with the result that some of the standardised values (E(S)) are negative. For ease of comparison of the standardised values an arbitarily chosen value of 5 has been added to all the standardised E indices so that they are all positive (E(S+5) Table 4). The high values of E index denote strongly lineated networks where junction bifurcation gives rise to a tributary sub-system of minimum magnitude with respect to the main stem. Conversely the low values of E index denote strongly compact networks where junction bifurcation gives rise to a sub-system of maximum value with respect to the main stem. The standardised values of E index (E(S+5)) in Table 4 show that the Grassridge and Oukloof systems are the most compact, while the Prinsrivier and Kammanassie systems are the most lineated. A visual examination of the drainage patterns supports this conclusion.

If the hypothesis that the exterior growth elements of the Sundays River system are more dynamic with respect to the internal elements as compared with the other systems is true, then the E indices (E(S+5)) for the Van Ryneveld's Pass Dam and the Lake Mentz networks would be smaller than those for the other systems. An examination of the values in Table 4 shows that this is not the case and the hypothesis must be rejected as invalid. If the hypothesis is rejected there is no evidence, in terms of the E index, to support the premise that the Sundays River system is adjusting in accordance with a shift in steady state conditions. However, the Sundays River system deviates so markedly from the general relationship between the P ratio and the mean annual sediment yield that further research into the structure of the Sundays River system is required. For the purposes of the present study the Sundays River system must be excluded and the model for the estimation of sediment yield will be based on the high significant correlation found between the P ratio and mean annual sediment yield in the other thirteen river systems.

The formulation of the model

It has been shown that the relationship between the P ratio and the mean annual sediment yield per unit area is linear within the limits of the available data (Figure 8) and therefore a linear regression of one variable on the other would provide the best mathematical equation to represent the relationship and to form the basis of a model for predicting mean annual sediment yield from measured values of the P ratio. However, a significant correlation between two variables does not imply that there is a physical relationship between them, merely that they vary in parallel with one another. Since there is no evidence to show that a finite change in one of the variables would be the direct cause of a finite change in the other, the use of regression analysis for the formulation of the model might appear to be suspect. Nevertheless, it is common practice to use regression analysis to express a relationship in mathematical form even when there is no direct cause and effect, and in such cases, the regression equation has no physical interpretation and can only hold true under specified conditions. In their introduction to applied regression analysis Draper and Smith (1966, p 2) state that, "Even where no sensible physical relationship exists between variables, we may wish to relate them by some sort of mathematical equation. While the equation may be physically meaningless, it may nevertheless be extremely valuable for predicting the values of some variables from knowledge of other variables perhaps under certain stated restrictions". Regression analysis can therefore be used to formulate the model but the model would only give satisfactory results under the conditions stipulated in the hypothesis, that is, under conditions of steady state and compliance with Horton's first two laws of drainage composition.

With the uncertainty regarding physical dependence between the two variables the choice of the independent variable for the regression analysis inevitably becomes arbitary. One of the assumptions involved in regression analysis is that the independent variable contains no stochastic component due to errors in measurement with the result that it is usual to make the prediction variable (the measured variable) the independent variable. Using Horton's P ratio as the independent variable a regression of mean annual sediment yield on the P ratios for the thirteen systems (correlation of 0,99 significant at the 0,95 level) gave the following regression equation;

$$S = 1{,}41P - 0{,}76 \qquad \text{(Eqn 11)}$$

where P is Horton's P ratio for the channel network (dimensionless) and S is the mean annual sediment yield per square kilometre of catchment measured in thousands of cubic metres (wet sediment). In the above equation a P ratio of less than 0,54 would result in a negative estimate of the mean annual sediment yield and because there is always erosion, the model must take the form:

$$\left.\begin{array}{lll} P \geqslant 0{,}54 & S = 1{,}41P - 0{,}76 & 10^3 m^3/km^2/yr \\ P < 0{,}54 & S < 0{,}001\ . & 10^3 m^3/km^2/yr \end{array}\right\} \qquad \text{(Eqn 12)}$$

As yet no P ratios smaller than 0,54 have been encountered in South African catchments, even in areas that are considered to have a negligible sediment yield (Roberts, 1974).

An estimate of the mean annual sediment yield derived by inserting the measured value of the P ratio into the model (Equation 12) should include the associated prediction errors which can be calculated in the form of 95% confidence limits. The confidence limits about the estimate of the sediment yield can be calculated from the formula given by Draper and Smith (1966, p 24);

$$\hat{Y} \pm t(v,\ 0{,}975)\sqrt{1 + \frac{1}{n} + \frac{(X_k - \bar{X})^2}{(X_i - \bar{X})^2}}\quad z \qquad \text{(Formula 1a)}$$

where $\hat{Y}$ is the estimated dependent variable obtained by inserting the measured value (X_k) of the independent variable in the regression equation. v is the number of degrees of freedom (n - 2) and z is the square root of the estimate of the variance about regression (z^2 or σ_{yx}^2), based on n - 2 degrees of freedom. The value of z is obtained from the equation,

$$z = \sqrt{\frac{SS}{n - 2}} \qquad \text{(Eqn 13)}$$

where SS is the sum of squares about regression and is given by;

$$SS = \Sigma\, Y_i^{\,2} - \frac{(\Sigma\, Y_i)^2}{n} - \frac{[\Sigma\, X_iY_i - (\Sigma\, X_i)(\Sigma\, Y_i)/n]^2}{[\Sigma\, X_i^2 - (\Sigma\, X_i)^2/n]}\,. \qquad \text{(Eqn 14)}$$

In Formula 1a and Equation 14, X_i and Y_i represent the individual observations of the P ratio and mean annual sediment yield respectively and by solving Equation 14 a value of 0,0096 was obtained for SS. This value was inserted into Equation 13 to obtain a value of 0,0295 for z. The value of z together with the value of t(11, 0,975) which was obtained from tables (Freund and Williams, 1958, p 502) can be included in Formula 1a to express the confidence limits in the form;

$$S \pm 2{,}201\sqrt{1 + \frac{1}{13} + \frac{(P - 0{,}669)^2}{0{,}172}}\quad 0{,}029 \qquad \text{(Formula 1b)}$$

or

$$S \pm 0{,}0638\sqrt{1{,}0769 + \frac{(P - 0{,}699)^2}{0{,}172}} \qquad \text{(Formula 1c)}$$

where the estimate of the mean annual sediment yield (S) has been substituted for $\hat{Y}$ and Horton's P ratio (P) has been substituted for X_k in Formula 1a. The curved 95% confidence bands derived by using Formula 1c have been drawn in Figure 8.

In summary, the model for the estimation of mean annual sediment yield in ungauged catchments takes the form;

$$\begin{aligned} P &\geqslant 0{,}54 \quad S = 1{,}41P - 0{,}76 \pm \varepsilon \quad 10^3 m^3/km^2/yr \\ P &< 0{,}54 \quad S < 0{,}001 \quad 10^3 m^3/km^2/yr \end{aligned} \qquad \text{(Eqn 15)}$$

where ε represents the 95% confidence limits about S, calculated by inserting the measured value of Horton's P ratio (P) in the equation

$$\varepsilon = \pm\, 0{,}0638 \sqrt{1{,}0769 + \frac{(P - 0{,}699)^2}{0{,}172}} \; . \qquad \text{(Eqn 16)}$$

The performance of the above model will be assessed in the following chapter by applying it to the catchment areas feeding the St Lucia Lake system which lies outside the areas used for the calibration of the model. The estimates of mean annual sediment yield will be compared with estimates derived by using existing empirical methods and the comparison will serve to illustrate the advantages and disadvantages of the model.

CHAPTER VII

CONCLUSION

The main problems that have been encountered in the formulation of the model for the prediction of mean annual sediment yield can be attributed to two principal factors which are firstly, the general lack of adequate measurements of long term sediment yield in South African catchments and secondly, the poor correlation that exists between sediment yield and any one or more of the easily measured variables that can be considered to influence the rate of erosion. It was the lack of suitable prediction variables that led to the examination of the seasonal pattern of variation in sediment yield. The variations in sediment yield have been explained in terms of the complex interaction of catchment surface variables and it was reasoned that the climate, vegetation and soil variables involved control not only the rate of erosion, but also the morphology of the channel network and the level of the morphoclimatic equilibrium that is realised in a drainage basin. Consequently the concept of a drainage basin as an open system in steady state can be used to link sediment yield with the morphology of the channel network on the premise that network morphology and sediment yield are controlled by the interaction of variables that give rise to steady state conditions and that any shift in the steady state will be accompanied by sympathetic changes in both network morphology and sediment yield. If the network morphology is controlled by the same conditions that control sediment yield, the choice of a suitable drainage network variable circumvents the problem of trying to find a suitable prediction variable amongst the large number of variables that have a direct influence on the sediment yield. Horton's P ratio was chosen as a variable representative of the channel network morphology and by using the available data on mean annual sediment yield it was possible to show that there is a high correlation between the P ratio and sediment yield. The resulting relationship between the P ratio and the mean annual sediment yield per unit area was used to formulate a model for the prediction of sediment yield in ungauged catchments.

In the introductory chapter it was stressed that there is a need for an additional method of estimating mean annual sediment yield to complement estimates derived from existing empirical methods. Before assessing the advantages and disadvantages of the model formulated in this study, the model will be applied to a typical 'problem' area with the twofold purpose of illustrating the wide range of estimates that result from the application of the available methods and secondly to compare estimates given by the P ratio method with the estimates derived by other investigators. The catchment area chosen for a test application of the P ratio method should lie outside the catchment areas used for the calibration of the model and it should be preferably an area in which the problem of sediment accumulation has been studied in some detail by other researchers. The St Lucia Lake system in Natal meets the above requirements and has been chosen as a suitable area.

An extensive investigation of the St Lucia Lake system was conducted during the period 1964 - 1966 by the 'Commission of Enquiry into the Alleged Threat to Animal and Plant Life in St.Lucia Lake' (1966). It is alleged that the large volumes of sediment carried by the rivers feeding St Lucia Lake (Figure 12) have materially reduced the depth of the lake and that the sediment has been detrimental to the floating vegetation and benthonic fauna. In their estimate of the sediment load carried by the rivers feeding the lake, the Commission found that there were insufficient measurements of sediment load to make direct calculations feasible and therefore they resorted to using measurements of sediment concentration in other Natal rivers. The Commission concluded from the sediment concentrations in Table 5 that the average concentration of the rivers feeding St Lucia Lake was unlikely to exceed 0,5% (by volume) as a maximum and they applied the figure of 0,5% to the mean annual runoff of the rivers to obtain a mean annual sediment yield of 2,284 10^6 m^3 for the system. In a subsequent study of the St Lucia Lake system, Hutchison and Pitman (1973) improved the estimates of the mean annual runoff for each river by using a digital simulation model and applied the figure of 0,5% to the improved estimates of mean annual runoff to obtain a mean annual sediment yield of 1,59 10^6 m^3 for the system. Hutchison and Pitman presented this estimate of the mean annual sediment yield with estimates derived by applying the methods of Midgley (1952) and Schwartz and Pullen (1966) (Table 6).

An independent estimate of the mean annual sediment yield for the St Lucia Lake system was obtained by calculating Horton's P ratio for each of the major rivers feeding the lake (Table 7). No adjustment was made to the estimates of mean annual sediment yield to allow for trap efficiency because high flows entering the lake are dispersed through the swamps where the heavy sediment is deposited and the fine sediment is flocculated by the salt water in the lake[18] with the result that the lake can be regarded as a reservoir with a 100% trap efficiency. The estimate of the mean annual sediment accumulation in the St Lucia Lake, derived from the P ratio method, has been listed in Table 8 together with the estimates derived by other researchers. The various estimates can be assessed by reference to the known present volume of sediment in the lake in conjunction with the estimated age of the bottom (earliest) sediments. The Commission (1966) calculated from depth recording of sediment that 3 730 10^6 m^3 of sediment has been deposited in the lake system and they concluded from Carbon 14 and Palaeontological dating methods that the age of the bottom sediment falls in the range of 1 000 to 4 000 years. If it is assumed that the rate of sediment accumulation in the lake has remained reasonably constant, the volume of sediment in the lake can be used to estimate the time that it would take to deposit the present volume at the rate predicted by each of the methods in Table 8 (Table 8 Column 4). The methods can now be assessed in view of the assumption that sedimentation in the lake began somewhere between 1 000 and 4 000 years ago.

Figure 12 The Rivers Feeding the St Lucia Lake System

TABLE 5

THE SEDIMENT LOAD OF SELECTED RIVERS IN NATAL

River	Silt content by weight : %			Silt content by volume: % *
	Average for 1955-56	Average for 1957-58	Mean for two periods	
Tugela at Bergville	0,10%	0,07%	0,09%	0,13%
Tugela at Colenso	0,12%	0,106%	0,11%	0,18%
Tugela at Mandini	0,28%	0,13%	0,23%	0,35%
Buffalo near Ingogo	-	-	-	0,57%
Pongola at Gollel	-	-	-	0,16%

* These values were calculated on the basis that the dry weight of deposited silt is 50 lbs per cubic foot and that the suspended load is increased by 25% to allow for bed load.

After the Commission of Enquiry into the alleged threat to animal and plant life in St Lucia Lake (1966).

TABLE 6

ESTIMATES OF THE SEDIMENT YIELD OF RIVERS FEEDING ST LUCIA LAKE ($10^6 m^3$ per annum)

River Catchment	Hutchison and Pitman	Midgley	Schwartz and Pullen
Mkuze	1,06	0,87	2,45
Mzinene	0,12	0,12	1,00
Hluhluwe	0,25	0,27	1,40
Nyalazi	0,16	0,20	1,05
* Pongola	-	3,15	2,83
* Mfolozi	3,73	3,10	3,45
Total Annual Sediment	1,59	1,46	5,90

* Do not flow into St Lucia lake and not included in total sediment.

After Hutchison and Pitman (1973, p 85).

TABLE 7

P RATIOS AND ESTIMATED SEDIMENT YIELDS OF THE RIVERS IN THE ST LUCIA SYSTEM.

River	P Ratio	Sediment yield $10^3 m^3/km^2/yr$	Catchment area km^2	Sediment volume $10^6 m^3/yr$	Confidence 95% range
Mkuze	0,70	0,23	4650	1,069	0,762-1,376
Mzinene	0,65	0,16	705	0,112	0,066-0,159
Hluhluwe	0,69	0,21	1030	0,216	0,148-0,284
Nyalazi	0,75	0,30	710	0,213	0,165-0,260
Pongola *	0,67	0,18	7870	1,416	0,897-1,936
Mfolozi *	0,73	0,27	10085	2,723	2,057-3,388
Total sediment for system				1,61	1,141-2,079

* Do not flow into the St Lucia system but have been included for comparison.

TABLE 8

A COMPARISON OF ESTIMATES OF SEDIMENT YIELD FOR THE ST LUCIA SYSTEM

Estimate No.	Method of estimation	Mean annual sediment yield $10^6 m^3$	No. of years taken to deposit present volume
1	Schwartz and Pullen	5,90	632
2	Commission of Enquiry	2,28	1635
3	P Ratio method	1,61	2317
4	Hutchison and Pitman	1,59	2346
5	Midgley	1,46	2555
	Average of 3, 4, 5	1,55	2406

An examination of Table 8 shows that in terms of sediment age the estimate derived by using the method of Swartz and Pullen (1966) can be regarded as excessive. The remaining estimates fall within the limits set by age determination of the sediment and it is interesting to note that there is close agreement between the estimate derived from the P ratio method and the methods of Hutchison and Pitman (1973) and Midgley (1952). The average of the estimates derived from these three methods is approximately 1,6 10^6 m^3 per annum which can be regarded as the mean annual sediment yield of the St Lucia Lake system. If the 95% confidence limits are calculated for the estimate derived by using the P ratio method, the resulting range of 1,14 to 2,08 10^6 m^3 for the system gives an accumulation time of between 1 800 and 3 300 years which falls within the limits set by age determination. The P ratio method differs from the other methods in that it is based on an analysis of the actual drainage networks in the problem area and not on regionalised parametres. As a result the mean annual sediment yields per unit area that are derived for each river network give a more reliable indication of which sub-catchments have the highest erosion rates. For the St Lucia system an examination of Table 7 shows that the Mkuze river contributes the largest volume of sediment because of its size, but the sediment yield per unit area is highest in the Nyalazi catchment and it is in the latter catchment that corrective measures would be the most effective.

The estimation of the mean annual sediment yield for the St Lucia Lake system has served to illustrate three main points, firstly that there is often a wide discrepancy between estimates derived from different methods, secondly that the P ratio method has provided an acceptable estimate in a test case and thirdly, that methods involving the use of flow values have the disadvantage that the errors inherent in the estimation of the flow are included with the errors associated with the estimation of the sediment concentration. The last point is clearly illustrated by the discrepancy between the estimate derived by the Commission (1966) and the estimate derived by Hutchison and Pitman (1973) where the same sediment concentration was used in both cases (Table 8).

In an overall assessment of the P ratio method of estimating sediment yields, it is necessary to emphasize two factors that are likely to limit the general application of the model. The first factor is that the application of the model requires a basic working knowledge of network morphometry and the second is that the efficient operation of the model is subject to the constraint that the drainage basin conforms to steady state conditions as well as Horton's first two laws of drainage composition. Deviation from the law of stream numbers or the law of stream lengths can be detected during the calculation of the bifurcation and length ratios, but as there is no ready method of detecting deviation from steady state, it is necessary to assume steady state conditions when applying the model. On the other hand, the model has a number of advantages, the most prominent being that it may be applied at any specific point in a drainage network by analysing the map plan of the network above the point at any convenient scale. The model is best suited to the estimation of sediment accumulation in proposed reservoirs because the use of reservoir sediment data for the calibration of the model makes some allowance for compaction of the sediment and the loss of the colloidal fraction. Another advantage of the model is that no knowledge of the flow regime is required which obviates the possibility of introducing additional errors.

It should be noted that the P ratio method of estimating sediment yield is not intended as a replacement of existing empirical methods, but should be used in conjunction with present methods to improve the estimate of the mean annual sediment yield. The analysis has shown that the problems presented by the lack of easily measured variables that have a direct influence on the sediment yield can be overcome, to a large extent, by adopting a "black box" approach to model formulation and that at least one useful prediction variable may be derived from the composition of the drainage network. The collection of additional data relating to sediment movement in South African catchments in future years will facilitate the improvement of the P ratio model and will provide a more sound basis for the investigation into factors controlling the production of sediment. It is hoped that the results obtained in this study will stimulate interest and further research into the morphometric approach to the estimation of mean annual sediment yield in ungauged catchments.

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APPENDIX A

RESERVOIR CATCHMENTS

The fifteen reservoir catchments used as a basis for the development of the model, cover a wide range of vegetational, geological and climatic conditions (Table 9). The river systems have been ranked from high to low in order of sediment production so that the relationships between sediment yield and the variables listed in Table 9 may be readily assessed. The most interesting features that emerge from Table 9 are:

(i) There is no evidence of a decrease in the range of sediment production with increasing catchment area as described for the United States of America by Brown (Appendix B). In fact, the calculated correlation coefficient for the relationship is + 0,06.

(ii) Sediment yield is at a maximum in catchments that have a mean annual precipitation of between 200 and 350 millimetres. This pattern corresponds with the pattern of variation in the United States of America as described by Langbein and Schumm (1958) who explain the generally high erosion rates of this group in terms of the relationship between vegetation and rainfall.

(iii) The eight reservoirs with the highest sediment yields (Lake Arthur to Tierpoort inclusive) are situated in the Karroo System of rocks (mainly sandstones and shales) and are covered by False Karoo or Pure Grassveld types of vegetation.

(iv) The seven catchments with lower sediment yields (Albasini to Schweitze-Reneke inclusive) have developed on geological strata other than the Karroo System (except for a small portion of the Loskop catchment) and are predominantly covered by False Sclerophyllous Bush and False Grassveld types of vegetation.

(v) The mean annual rainfall and range of precipitation over the Prinsrivier, Oukloof and Kammanassie catchments should, in view of the relationships described by Langbein and Schumm (1958), result in relatively high sediment yields. The lower yields for these catchments may be ascribed to the more resistant strata of the Cape System (as compared to the Karroo System) and to the differences in vegetation type.

(vi) The percentage water yield (mean annual runoff expressed as a percentage of mean annual rainfall volume) which provides a measure of the effectiveness of mean annual rainfall, does not bear any relationship to sediment yield. It may be assumed therefore that the erodibility of the rocks and the vegetation cover have stronger influence on sediment production than the water yield from the catchment.

TABLE 9

THE RESERVOIR CATCHMENTS

Reservoir Name	River	River System	Catchment area km^2	Mean annual sediment yield $10^3 m^3/km^2/yr$	Mean annual Precipitation (mm)	Variation of annual Precipitation over catchment	Water yield %	Vegetation type	Geological system
Lake Arthur	Tarka	Gt. Fish	5879	0,605	419	1000-300	3	D(E)	K3
Grass Ridge	Gt. Brak	Gt. Fish	4099	0,412	357	500-300	3	D	K3
Van Ryneveld's Pass	Sundays	Sundays	3825	0,303	378	500-300	2	E(D)	K3
Lake Mentz	Sundays	Sundays	12491	0,276	315	600-200	4	E(D,C)	K3
Egmont	Witspruit	Caledon	326	0,260	643	700-590	6	E	K4
Kommando Drift	Tarka	Gt. Fish	3517	0,210	437	1000-300	3	D(E)	K3
Koppies	Rhenoster	Vaal	2196	0,198	628	650-600	8	E	K3,(K2)
Tierpoort	Kaffir	Riet	940	0,156	548	550-500	4	E	K3
Albasini	Luvuvhu	Pafuri	510	0,135	806	1250-500	9	A(B)	AG(DR)
Kammanassie	Kammanassie	Gouritz	1631	0,103	463	1100-200	5	G	C1(C2)
Olifants Nek	Hex	Crocodile	489	0,089	689	700-650	4	F	T3
Prins-rivier	Elands-Kloof	Gouritz	761	0,088	321	500-100	1	G	C1(C2,C3)
Oukloof	Cordiers	Gouritz	147	0,038	455	1200-200	5	G	C1(C2,C3)
Loskop	Olifants	Olifants	12281	0,011	732	750-650	5	F(B)	W(T3,K2,L,LF)
Schweitze-Reneke	Hartz	Hartz	8935	0,009	526	550-475	1	B(E)	V(I)

NOTE: Codes in brackets represent subsidiary types. KEY: See page 80

TABLE 9 (Continued)

KEY: Explanation of the Vegetation Symbols

A	Inland Tropical Forest Types
B	Tropical Bush and Savannah Types
C	Karoo and Karroid Bushveld Types
D	False Karoo Types
E	Pure Grassveld Types
F	False Grassveld Types
G	False Sclerophyllous Bush Types

Source: Acocks, J.P.H., 1951: Veld Types of South Africa, Pretoria, Government Printer.

Explanation of Geological Symbols

K2	Ecca Series	Karroo System
K3	Beaufort Series	
K4	Stormberg Series	
C1	Table Mountain Series	Cape System
C2	Bokkeveld Series	
C3	Witteberg Series	
T3	Pretoria Series	Transvaal System
W	Matsap Beds	Waterberg System
V	Andesitic lava	Ventersdorp System
I	Kalahari Beds	Tertiary to Recent
L	Gamagara Series	Loskop System
LF	Rooiberg Felsite	Bushveld Igneous Complex
DR	Andesitic lava	Dominion Reef System
AG	Granitic gneiss	Archaeon Complex

Source: Geological Survey, 1970: Geological Map of the Republic of South Africa and the Kingdoms of Lesotho and Swaziland, Department of Mines.

APPENDIX B

EMPIRICAL METHODS OF ESTIMATING MEAN ANNUAL SEDIMENT YIELD IN UNGAUGED CATCHMENTS

The main problems encountered in the derivation of a model for the estimation of mean annual sediment yield in ungauged catchments were outlined in Chapter I as being the general lack of reliable data for calibrating the model and the lack of suitable prediction variables. Regular trigonometric surveys of large reservoirs provide the most reliable source of data for model calibration (Schwartz and Pullen (1966) and Flaxman (1972)), but the number of large reservoirs is limited and to increase the volume of data, models have been based on suspended sediment data as well as reservoir data (Midgley (1952)) or on suspended sediment data alone (Anderson (1954) and (1970), Task Committee on Preparation of Sediment Manual (1970), Williams and Berndt (1972), Doornkamp and Tyson (1973), Yamamoto and Anderson (1973), Jansen and Painter (1974) and McPherson (1975)).

The estimates of mean annual suspended sediment load used to calibrate models have been derived from measurements of suspended sediment concentration. The derivation of the estimates necessitates a determination of the relationship between suspended sediment concentration and discharge, and the determination of this relationship provides one of the major sources of error in estimating the long term average yield. A plot of suspended sediment concentration against discharge (commonly known as a sediment rating curve) usually results in so wide a scatter of points that the relationship cannot be determined with any confidence. A number of techniques have been devised to improve the plot of concentration against discharge. Methods of smoothing a rating curve have been described by Miller (1951) as;

(i) plotting suspended sediment concentration against discharge on double logarithmic paper,

(ii) plotting cumulative suspended sediment concentration against cumulative discharge, and

(iii) plotting separate rating curves for each season of the year. Roberts (1952) found that adequate approximations of the sediment rating curve could be derived by taking the average of suspended sediment concentrations for equal or nearly equal discharge. The average values were then plotted against discharge to obtain a smoothed sediment rating curve. Sediment rating curves can be used to estimate mean annual suspended sediment yield by:

(i) plotting a duration of river flow curve for the river at the sampling site (discharge plotted against the percentage of time that it is equalled or exceeded),

(ii) transforming the river flow duration curve to a silt flow duration curve by referring to the sediment rating curve,

(iii) measuring the area below the river flow and silt flow duration curves to obtain the mean annual runoff and the mean annual silt flow respectively (Roberts (1952)).

Nordin and Sabol (1973) have shown that where a linear relationship between annual stream flow and annual suspended sediment load can be developed, the relationship can usually be used to extend the record of suspended sediment concentrations to cover the full period of the stream flow record. The extension of the sediment record often provides a better estimate of the mean annual suspended sediment load provided that the relationship between annual sediment discharge and annual water discharge remains the same for the entire period of stream flow record. Nordin and Sabol (1973) used the relative information content of the mean (defined as the ratio of variance of the mean for a random series of annual sediment loads to the variance of the mean for a non-random, or autocorrelated, series of annual sediment loads) as a criterion to determine whether or not estimates of long term suspended sediment yields can be improved by estimating sediment loads from the linear relationship. In the estimation of mean annual suspended sediment loads it is apparent that there are three major sources of error, namely;

(i) those involving the measurement of suspended sediment concentration,

(ii) those involving the estimation of long term average river flow, and

(iii) those resulting from the smoothing of the sediment rating curve.

In view of the sources of error and the fact that some estimate of bed load must be added to the estimated mean annual suspended sediment yield, estimates of mean annual sediment yield drawn from reservoir surveys are more reliable than estimates drawn from suspended sediment sampling. Where possible reservoir survey data therefore should be used for the calibration of models to estimate sediment yields in ungauged catchments. In view of these considerations, reservoir data were used for the derivation of the model in Chapter VI.

The choice of suitable prediction variables for use in a model is complicated by the fact that the sediment yield of a river system is influenced by a very large number of interrelated variables with the result that no single variable has been found to explain a sufficiently high proportion of the variation in sediment yields. To overcome the problem of the relatively low correlation between sediment yield and single prediction variables, a number of models have been based on multilinear regression equations using selected geomorphic, hydrologic and climatic variables that might be expected to influence sediment yield. The variables commonly used in multilinear regression equations could be summarised as being basin elevation, basin relief, basin slope, basin area, main channel length, mean annual precipitation, mean annual temperature, unit mean discharge and soil erodibility indices. (Anderson (1954), Task Committee on Preparation of Sediment Manual (1970), Flaxman (1972), Williams and Berndt (1972), Jansen and Painter (1974), and McPherson (1975)). In addition several indices have been devised to approximate the erodibility of rocks and vegetation density (Jansen and Painter (1974)) and types of land use (Anderson (1954), Task Committee on Preparation of Sediment Manual (1970) and Williams and Berndt (1972)).

A discussion of the methods of Jansen and Painter (1974) and McPherson (1975) will illustrate some of the disadvantages of the multilinear regression approach to the estimation of sediment yield, and will provide an insight to the suitability of the models for use in South African catchments. These two models have been developed recently and were chosen for discussion because it was felt that the techniques used and the variables selected are representative of the multilinear regression approach adopted in other countries.

The method of Jansen and Painter (1974).

Jansen and Painter (1974) used multilinear regression models to relate average annual suspended sediment yield to climatic and topographic variables. Estimates of mean annual suspended sediment yield were obtained for seventy-nine catchments in twenty-eight countries and the catchments selected represent all of the world's major climatic zones. The catchments were grouped according to their climatic zone by using the A-D classification of Trewartha (1943). A multiple regression analysis was conducted for each climatic zone between the average annual suspended sediment yield and logarithmic transformations of a number of catchment variables that could be measured in all the catchments. The eight variables that Jansen and Painter selected include:

G, a rock softness or proneness to erosion index where G was given a value of three for Palaeozoic rocks, five for Mesozoic rocks, six for Cainozoic rocks and two for Quaternary rocks, and

V, a vegetation index to approximate vegetation density where V was given a value of four for forest, three for grassland, two for steppe and one for desert.

The variables G and V appear to be very course approximations of rock softness and vegetation density and the values ascribed to them appear to have been optimised to give the best multiple regression coefficient for the catchment areas studied. The applicability of these values outside the study area is questionable as there is likely to be a wide range in proneness to erosion for rocks of any particular age as well as a wide range in vegetation cover in the four vegetation classes.

In each multiple regression equation the parameters that were not significant at the 90% level were eliminated to give one significant equation for each climatic zone (Jansen and Painter, 1974, p 376). In the case of C climates catchments only, the residuals showed a distinct grouping into catchments of high and low relief as denoted by the relief parameter R and therefore two regression equations were derived based on the value of R (Jansen and Painter, 1974, p 377). Finally a multiple regression equation was calculated for all the climates to give a general empirical relationship (Jansen and Painter, 1974, p 378). An examination of the regression coefficients in the equation for all climates indicates that mean annual suspended sediment yield has a direct relationship with runoff, altitude, relief, precipitation, temperature and rock softness whereas there is an inverse relationship with catchment area and vegetation cover. If the signs of the regression coefficients are accepted as representative of the nature of the relationship between the individual variables and sediment yield, there appear to be three anomalies in the other equations given by Jansen and Painter. In the equation for A climates there is an inverse relationship with temperature which Jansen and Painter ascribe to the fact that higher temperatures provide better vegetation cover in tropical climates. The inverse relationship with discharge in the equation for B climates has been attributed to high intensity floods that are characteristic of arid areas with low mean discharge, but no explanation could be found for the inverse relationship with relief in one of the equations for C climates. Jansen and Painter (1974, p 380) have stressed that "considerable care must be taken when such predictions are made for catchments outside the population on which the analysis was based", therefore the performance of the models should be tested with measured sediment yields from South African catchments before applying the method in the South African context.

The method of McPherson (1975).

The method derived by McPherson is based on multiple regression equations relating numerous geomorphic and hydrologic variables to suspended sediment yield and the yield of dissolved solids. The equations were developed for the purpose of estimating sediment yields in ungauged catchments and the yield of dissolved solids was included for erosion studies. The dissolved solids yield is of little consequence when estimating the rate of sediment deposition in reservoirs. The data were drawn from thirty-six drainage basins in Southern Alberta (Western Canada) where the basins include a diversity of climatic, geologic, vegetative and physiographic type. The geomorphic and hydrologic variables chosen by McPherson for regression with sediment yield were basin area, basin diameter, mean basin elevation, local relief, elongation ratio, mean annual discharge, mean land slope, main channel length, stream gradient and unit mean discharge. Both linear and logarithmic models were derived with each variable and the equation with the highest correlation coefficient was adopted.

McPherson found that there was no significant relationship between suspended sediment yield and any one of the geomorphic or hydrologic variables. However, the yield of dissolved solids correlated significantly with unit mean discharge, mean land slope, elevation, local relief, elongation ratio and stream gradient with the highest correlation coefficients being obtained for the first two of these variables. When the suspended sediment yield was added to the dissolved solids yield to obtain what McPherson refers to as total sediment yield (this does not include any measure of bed load) correlation coefficients in excess of

0,80 were obtained with elevation, local relief, mean land slope and unit mean discharge. McPherson concluded that the geomorphic variables of elevation and slope best explain the yield of dissolved solids and the total sediment yield.

By using multilinear regression, significant regression equations were obtained for suspended, dissolved and total sediment yield (McPherson, 1975, p 256). Estimates of mean annual suspended sediment yield could be derived for basins in Southern Alberta by applying the equation for total sediment load and adding some estimate for bed load, but there is little application for the equation outside the research area.

A major criticism of the use of multiple regression techniques for the prediction of sediment yield is that the independent variables used are often interdependent. For example, Jansen and Painter (1974, p 376) have used mean annual precipitation, mean annual discharge and vegetation density in the prediction equation for B climates; and McPherson (1975, p 256) has used basin diameter with mean land slope in the equations for dissolved solid yield and total sediment yield. In cases of multicollinearity where there are high correlations among independent variables, the estimates of the regression and partial correlation coefficients tend to have large standard errors (King, 1969, p 162) with the result that the significance of the equations is often in doubt. It is felt that stated significance levels for multilinear regression equations can sometimes lead to a false sense of confidence in the derived estimates of sediment yield. For example, Flaxman (1972) obtained a multiple correlation coefficient of 0,958 for a prediction equation that explained 92% of the variation in sediment yields. The F value greatly exceeded the 1% level of significance but when computed sediment yields were plotted against measured sediment yields a wide scatter of points resulted for the smaller catchments and six of the thirty-nine computed values were negative (Flaxman, 1972, p 2081). A number of problems associated with multiple regression equations could be avoided by using the principle components approach in prediction equations for sediment yield, as described by Anderson (1970) and Yamamoto and Anderson (1973). "The advantages of this approach over stepwise or full scale regression has been amply demonstrated in (1) obtaining stable evaluation of predictive coefficients (2) in identifying important variables and (3) in evaluating physical factors contributing to explained variance ..." (Anderson, 1970, p 412). However, as McPherson (1975, p 253) has pointed out "... the greatest problem with models developed so far, is that they have generally proved not to be sufficiently robust to be transferable from one hydrologic region to another".

The methods that have been derived for the estimation of mean annual sediment yield in South African catchments (Midgley (1952), Schwartz and Pullen (1966) and Doornkamp and Tyson (1973)) contrast with the multilinear regression models discussed above. The South African methods have been designed to depict the general variation of sediment yield throughout the country and use maps or graphs from which an estimate of the sediment yield may be obtained for a catchment by reference to its area and geographical location. The methods, applicable only to South African catchments, are based on single prediction variables and the delimitation of areas with similar sediment producing characteristics.

The method derived by Midgley (1952).

The first comprehensive guide to the estimation of mean annual sediment yield in ungauged catchments in South Africa was a silt map compiled by Midgley (1952). The silt map formed part of an unpublished thesis and its availability was limited until it was reproduced recently in a publication by Doornkamp and Tyson (1973).

At the time of compilation of Midgley's silt map, periodic silt surveys of fifteen reservoirs in South Africa were available. Measures of mean annual sediment yield were derived from these surveys and Midgley adjusted the sediment yields for loss of silt over spillways by applying a trap efficiency correction (MacKenzie, 1949, Appendix D) for each reservoir. In view of the fact that the mean annual sediment yields derived from the reservoir surveys represent the total sediment load, the validity of Midgley's use of a suspended sediment trap efficiency index may be questioned.

Sampling of the suspended sediment concentration of river flow had been undertaken since 1929 and concomitant flow records were available for forty-eight stations. Midgley plotted the measured percentage of silt (by weight) against the discharge at the time of sampling and drew a smoothed curve through the points for each station. Flow duration curves showing the percentage of time that any flow is equalled or exceeded were then drawn for each sampling point and the duration of river flow curves were converted to duration of silt flow curves by making use of the relevant percentage silt/discharge curves. In order to facilitate the conversion, the percentages of silt by weight were transformed to percentages of silt by volume by assuming that the average expected density of silt when deposited in a reservoir would be fifty pounds per cubic foot. The choice of the value of fifty pounds per cubic foot is not unrealistic in view of the fact that reported volume weight values for reservoirs in the United States of America vary from twenty pounds per cubic foot (320,3kg/m^3) to one hundred and ten pounds per cubic foot (1 761,6kg/m^3) (Megahan, 1972, p 1335). The duration of river flow and the duration of silt flow curves were integrated to give estimates of the mean annual runoff and the mean annual silt flow respectively, and the quotient gave the long term average silt concentration by volume at the sampling point. An allowance for bed load was made by adding between 25% and 50% of the mean annual suspended sediment load, where the percentage of bed load was given a high value for rivers with a relatively steep gradient. Having obtained a measure of the mean annual sediment yield expressed as a percentage of the river flow at a number of locations, Midgley then divided the country into zones for which the sediment yield as a percentage of the river flow would be reasonably uniform. The zones were delimited by making use of a soil erosion map (Department of Irrigation) and by reference to the vegetation and geology. Using the derived silt percentages it was possible, by analogy, to assign silt percentages to similar areas and the application of the silt percentages to the mean annual runoff for each area gave the mean annual sediment yield which was marked on the silt map.

The delimitation of zones having reasonably similar sediment producing characteristics necessitates a large measure of subjectivity and when using Midgley's silt map cognisance should be taken of the fact that the range of sediment yields in each zone has been derived, to a large degree, by reference to sediment yields calculated from suspended sediment concentrations. The very poor relationship that exists between suspended sediment concentration and discharge (as discussed in Chapter II) makes the estimation of long term yield from short term suspended sediment records subject to large errors. Nevertheless, the map provides a very useful source of a first estimate of mean annual sediment yield for any catchment in South Africa.

The method of Schwartz and Pullen (1966).

An alternative method of estimating mean annual sediment yield was produced by Schwartz and Pullen (1966) and is based on the observation of Brown (Trask, 1950) that the range of sediment production in the United States of America tends to narrow with increase in catchment size. The narrowing in range of sediment production with increasing catchment size is attributed by Schwartz and Pullen to the greater influence of local factors such as gradient, soil type, vegetation cover and land use in small catchments. However, there is also the consideration that the probability of complete coverage by individual high intensity storm events is much higher for small catchments with the result that a higher percentage of the catchment area reacts to storm events. A plot of sediment yield per unit area against catchment area usually results in a wide scatter of points as the relationship is poorly defined. For example, the general inverse relationship between sediment yield per unit area and catchment area is not evident in the data (Table 9) used for the derivation of the model in Chapter VI. In view of the poor relationship between the two variables, the choice of catchment area by Schwartz and Pullen (1966) as the dominant criterion for estimating sediment yield limits the confidence that can be placed in derived estimates.

Schwartz and Pullen made use of mean annual sediment yields derived from reservoir surveys of fifteen reservoirs in South Africa. The sediment yields were marked on a map at their appropriate geographical locations and five extensive zones based on these values and considerations of general topography, climate, geological features and veld type were delineated to represent zones of equal index of silt production. The zones were graded from one to five on the basis of their level of silt production. The values of mean annual sediment yield were then plotted on a double logarithmic paper against catchment area and five converging trend lines were drawn to represent the five zones (Schwartz and Pullen, 1966, Fig 1, p 344).

No precise description is given by Schwartz and Pullen of the criteria used to delimit the zones and as they divided the country into approximately one half of the number of zones used by Midgley it is apparent that the criteria used were not the same as those chosen by Midgley. There is some uncertainty about the validity of the delimitation of zones of equal index of silt production by Midgley (1952) and Schwartz and Pullen (1966) because a study by McPherson (1975) showed no clear pattern of variation of sediment yield according to physiographic region. In addition the method of Schwartz and Pullen provides a wide range of sediment yield for any one catchment and the range increases rapidly with decreasing area in any zone. A notable feature of the plot of catchment area against sediment yield (Schwartz and Pullen, 1966, Fig 1, p 344) is that the zone lines change from negative to positive slope from line 5 to line 1 and in view of the general inverse relationship described by Brown (Trask, 1950) the interpreted inversion of the relationship from one area to another needs justification. Mean annual sediment yields derived by using the method of Schwartz and Pullen tend to be greater than estimates derived from Midgley's silt map (for example, the estimates derived for the St Lucia Lake system, Chapter VII), and it is suggested that the discrepancy may result from the positive slope given to the lines for zones 1 and 2.

The method of Doornkamp and Tyson (1973).

A method of estimating mean annual suspended sediment yield was developed by Doornkamp and Tyson (1973) who made use of the equations of Fournier (1960) to produce maps depicting the variation of suspended sediment yield in South Africa. There is often some difficulty in obtaining an objective determination of Cm (defined on p 13) and as a result the Fournier equation is usually replaced by four relationships based on considerations of relief and rainfall (Doornkamp and Tyson, 1973, p 336). The four equations take the form of simple linear regression equations between suspended sediment yield and the precipitation variable p^2/P (Doornkamp and Tyson, 1973, p 336) and have been derived by subdividing the data into relief and rainfall categories. This approach usually results in higher correlations and better performance of the models, provided that there is still sufficient data in each category to obtain statistically significant results. The validity of the division of the data into relief categories is supported by the finding of Jansen and Painter (1974) that in Mesothermal climates the residuals of a multiple regression equation for the prediction of sediment yield showed distinct grouping into categories of high and low relief.

Doornkamp and Tyson (1973) applied the four simple regression equations to available rainfall data to produce two maps depicting the variation of suspended sediment yield; one for areas of little relief and one for areas of pronounced relief. The ratio of maximum mean monthly rainfall to mean annual rainfall (p^2/P) used by Fournier (1960) gives an index of seasonal rainfall concentration which should cruedly represent relative variation of vegetation cover and stream discharge. The variations in the resistance of geological strata are approximated by the relief categories so that the prediction equations generalise the climatic, geologic and vegetation variables in a catchment, but as pointed out by Doornkamp and Tyson, the effects of local controls may give rise to marked variations from the generalised relationships.

When the derived estimates of suspended sediment yield were compared by Doornkamp and Tyson (1973, p 339) with estimates obtained from Midgley's silt map it was found that there was a large measure of agreement. However, Doornkamp and Tyson (1973, p 337) are of the opinion that the Fournier (1960) equations tend to overestimate the amount of natural erosion and consequently the patterns of variation depicted by the two maps may be more valid than the values recorded.

It is of interest to note that other variables that could be expected to correlate with sediment yield such as drainage density, length of overland flow, constant of channel maintenance and stream frequency have not been included in the studies outlined above. Gottschalk (1964) for example, found in Central Dakota that drainage density was a significant factor influencing sediment production rates. One of the reasons why variables such as drainage density and stream frequency have been omitted may be that all stream channels in the basin must be measured which makes the use of the variables impractical in the general prediction equations, especially when the equation is to be used for large catchments. The disadvantage of using variables that are dependent on scale highlights the advantages gained by using Horton's P ratio which has been shown to correlate very highly with sediment yields in South Africa (Chapter VI) and can be measured easily from topographical maps at any convenient scale. As the P ratio model is based on the measurement of a single variable, it does not suffer from the same limitations as the other methods used in South Africa which are dependent largely on the delimitation of zones of equal index of silt production. The use of a single variable that correlates highly with sediment yield makes the P ratio model relatively simple to apply and overcomes the problem of multicollinearity in multiple regression models. If a single variable that correlates highly with sediment yield and is applicable to all South African catchments had not been found, a multilinear regression approach would have been attempted with the geomorphic, hydrologic and climatic variables used in the multilinear regression models described above.

APPENDIX C

THE CALCULATION OF HORTON'S P RATIO

The procedure adopted for the calculation of Horton's (1945) P ratio was described in Chapter VI and may be illustrated by using the examples of Van Ryneveld's Pass and Lake Mentz river networks. In Chapter VI it was stressed that as the bifurcation and length ratios that comprise the P ratio are intra-system ratios, maps at any suitable scale could be used for the derivation of the P ratio. In order to examine the extent to which scale affects the value of the P ratio in practice, Horton's P ratio for Van Ryneveld's Pass Dam and Lake Mentz networks were recalculated using a map at a different scale from that used to calculate the ratios in Table 1. The original map scale used for the Sundays River system was 1:1 000 000 and the bifurcation and length ratios have been recalculated from a topographical map at a scale of 1:500 000 to give the values listed in Table 10. The procedure can be summarised as follows:

(i) The networks were traced off the map and ordered according to the method of Horton (1945).

(ii) The number of streams of each order was noted and the systems were renumbered before measuring the total length of stream channel in each order with a map measure tracer wheel.

(iii) The total length of stream channel in each order was divided by the number of streams of that order (counted before renumeration) to obtain the average length of stream channel for each order.

(iv) The bifurcation ratio was obtained from the anti-logarithm of the regression coefficient for the relationship between stream order and logarithmic values of the number of streams in each order. (Logarithmic values were used to convert the geometric relationships observed by Horton (1945) to linear relationshps so that the ratios could be determined by regression.) Similary the length ratio was obtained by taking the anti-logarithm of the regression coefficient for the relationship between stream order and logarithmic values of average stream length in each order. The data for the main stream were not used in the calculation of the bifurcation and length ratios because the main stream extends beyond the reservoir.

(v) The P ratio was calculated by dividing the length ratio by the bifurcation ratio.

A comparison of the bifurcation, length and P ratios for the two systems in Table 1 and Table 10 shows that the use of a larger scale map has resulted in a small reduction of the bifurcation and length ratios in both networks. However, the values of the P ratio are similar, a fact which supports the premise that the value of the P ratio is not dependent on the scale of the map used. The correlation coefficient for all four regression equations was 0,99 (significant at the 0,95 level) demonstrating that Horton's first two laws of drainage composition hold for the Sundays River system. The P ratios for the other thirteen systems were obtained in the same way and it was found that Horton's two laws were obeyed in all the systems. The fifteen river systems cover a wide range of climatic, geologic and vegetative conditions (Appendix A) and as Horton's two laws hold for all fifteen systems it can be assumed that the laws hold for the majority of South African catchments and that the P ratio model is widely applicable in South Africa. Where this is not the case non-conformity with Horton's laws can be detected during the calculation of the bifurcation and length ratios and should the laws not hold in a catchment the applicability of the model can be questioned.

TABLE 10

P RATIO CALCULATION

Van Ryneveld's Pass Dam

Stream order	No. of streams	Log. no. of streams	Total stream length (units)	Average stream length (units)	Log. average stream length
1	180	2,2553	697,5	3,875	0,5883
2	40	1,6021	368,8	9,220	0,9647
3	9	0,9542	200,1	22,223	1,3468
4	3	0,4771	159,9	53,300	1,7267
5	1	-	-	-	-

rb = 3,965 (r = -0,997 and b = -0,5982)

rl = 2,397 (r = 0,999 and b = 0,3797)

P ratio = 0,6045

Lake Mentz

1	649	2,8122	2631,6	4,055	0,6080
2	152	2,1818	1491,5	9,813	0,9918
3	42	1,6232	1018,0	24,238	1,3845
4	9	0,9542	593,3	65,927	1,8191
5	2	0,3010	290,0	145,039	2,1614
6	1	-	-	-	-

rb = 4,217 (r = -0,999 and b = -0,6250)

rl = 2,474 (r = 0,999 and b = 0,3934)

P ratio = 0,5866

KEY:			
rb	bifurcation ratio	rl	length ratio
r	correlation coefficient	b	regression coefficient

APPENDIX D

THE TRAP EFFICIENCY CORRECTION INDEX

The values of mean annual sediment yield used for the calibration of the model in Chapter VI have been derived from reservoir surveys and an attempt was made to improve the performance of the model by making allowance for differences in trap efficiency from one reservoir to another. The model originally gave a correlation of 0,98 between Horton's P ratio and mean annual sediment yield without adjustment for trap efficiency, and by use of an optimised trap efficiency correction index this high correlation was even further increased. The methods of MacKenzie (1949) and Brune (1953) are most commonly used for the estimation of trap efficiency but they were regarded as unsuitable for application to the model because they only apply to suspended sediment volume and not to the total sediment volume. For this reason a trap efficiency correction index was derived.

When assessing estimates of trap efficiency it is important to remember that the percentage of incoming sediment trapped by a reservoir over a long period is dependent on a very large number of factors, including;

(i) the storage capacity of the reservoir in relation to the mean annual runoff of the contributing river or rivers,

(ii) the manner of regulation of outflow and orientation of spillway and outlets relative to inflow direction,

(iii) the reservoir stage at the time of arrival of floods,

(iv) the water temperature,

(v) the presence of salts in the water which might promote or inhibit flocculation and hence the rate of sedimentation,

(vi) the removal of exposed dry sediments by wind action at low reservoir stages, and

(vii) the shrinkage and compaction of silt (Midgley (1968)).

Although each of the above factors influences trap efficiency, a reasonable overall estimate of reservoir trap efficiency can be obtained by using the ratio of storage capacity to mean annual runoff (Midgley (1968)). This choice of the storage to inflow ratio can be justified by considering extreme values of the ratio. For example, a barrage-type structure (with storage space mainly confined between the original river banks) has a low storage to inflow ratio and would trap all of the bed load, but only a small percentage of the suspended load. On the other hand, a large reservoir of sufficient capacity to retain all of the inflow (high storage to inflow ratio) would trap the entire sediment load and have a trap efficiency of 100%. The capacity to inflow ratio can be used as a guide to suspended sediment trap efficiency if the ratios are plotted against measured trap efficiencies to provide a generalised curve.

The trap efficiency of a reservoir as applied to suspended sediment load only, can be approximated by taking concomitant measurements of the suspended sediment load entering the reservoir system and the suspended sediment load leaving the system by way of sluice gates, outlet pipes and spillway. The difference between the input and output volumes of sediment can be regarded as the volume of sediment retained. Values of suspended sediment trap efficiency obtained in this way have been plotted against storage to inflow ratio for forty normal-type reservoirs in the United States of America by Brune (1953). A similar trap efficiency curve was produced for South African conditions by MacKenzie (1949) who based the curve essentially on the measured trap efficiency of Vaal Dam. By using the origin and the single value of trap efficiency as two fixed points he drew a curve to conform with a quantity curve, that is, a curve "showing as an average over a long period, the probable quantity of water retained by dams of various relative capacities" (MacKenzie, 1949, p 5). Despite the fact that this curve is based on one value of trap efficiency, it has been extensively used in South Africa by Roberts (1952), Menne and Kriel

(1959), Midgley (1968) and by de Wet (1973, p 110) who expressed the relationship algebraically. Although the suspended sediment trap efficiency curves of both MacKenzie (1949) and Brune (1953) are based on the capacity to inflow ratio, they do not provide similar estimates of trap efficiency. When the two curves are plotted on the same axes (Midgley, 1968, Fig. 3.2, p 3.5) it illustrates the large difference in estimated trap efficiency. Midgley (1968) interpreted the difference between the two curves as indicative of the seasonal variations of river discharge, that is, a reservoir fed by a perennial river would have a greater trap efficiency as compared with a reservoir having the same capacity to inflow ratio, but fed by a river with irregular flow characteristics.

In the derivation of the model the measured sediment yields include the bed load fraction as well as suspended sediment load which prohibits the use of the above trap efficiency indices that have been based only on suspended sediment. It became necessary to derive a trap efficiency index that could be applied to total sediment loads, using data readily available for all thirteen reservoirs. In deriving a trap efficiency correction index the original capacity to inflow ratio was used as a reference and two basic assumptions were made, namely:

(i) That a reservoir with a capacity to mean annual runoff ratio of unity would have a trap efficiency of 100%. It was reasoned that if the reservoir could store all the incoming water in an average year in terms of runoff, then spillage would not occur very often and that in all but major floods the water passing over the spillway would be displaced water from the reservoir and not the heavier silt laden water entering the reservoir.

(ii) That for reservoirs with a capacity to mean annual runoff ratio of less than unity, the relationship between total trap efficiency and capacity to inflow ratio would be linear. In view of the exponential nature of the relationship between capacity to inflow

> ratio and suspended sediment trap efficiency as expressed by de Wet (1973, p 110), the validity of the assumption appeared suspect. Calculations, however, made it evident that a correction factor based on a non-linear function gave too much weight to the reservoirs with lower capacity to inflow ratios and thereby decreased the correlation coefficient and the performance of the model.

If it was assumed therefore that there is an inverse linear relationship between the trap efficiency of a reservoir with respect to total sediment load and full supply capacity expressed as a percentage of the mean annual runoff, the equation for the trap efficiency could take the form;

$$T = 100 - C \qquad \text{(Eqn 17)}$$

where

T is the trap efficiency correction index expressed as a percentage of the mean annual volume of accumulating sediment,

and

C is the full supply capacity expressed as a percentage of the mean annual runoff.

The above trap efficiency correction index has been designed to accommodate for the percentage volume of sediment that passed through the reservoir and should therefore be applied to the mean annual sediment yield expressed as an annual volume and not to sediment yield expressed as a volume per unit of catchment area.

Trap efficiency indices derived from Equation 17 were used to adjust the thirteen values of mean annual sediment yield. It was found that when the maximum possible correction was confined to 10% of the mean annual sediment volume,

$$T = \frac{100 - C}{10} \qquad \text{(Eqn 18)}$$

the correlation coefficient rose from 0,98 to 0,99. With a correlation of 0,99 it was felt that no further optimization of the trap efficiency correction index was required and the arbitary maximum limit of 10% was therefore not increased. It should be noted that the trap efficiency correction index given by Equation 18 is an optimised variable to improve the performance of a model for the estimation of mean annual sediment yield and therefore does not necessarily give a reasonable estimate of the true total trap efficiency at any reservoir.

APPENDIX E

ADDITIONAL NOTES

1. The problem of a large number of variables also applies to the estimation of discharge (Mustonen (1967)).
2. 'the total sediment load is usually subdivided' to read 'the total sediment load may be subdivided'.
3. See Appendix A.
4. See Appendix B.
5. 'sediment yield took the form of a silt map' to read 'sediment yield came from a silt map'.
6. 'the method takes the form of maps' to read 'the method makes use of maps'.
7. 'the individual values used' to read 'the indices used'.
8. This assumption is supported by the fact that sediment accumulates on the catchment surface during dry periods.
9. The steady state concept has been applied to the distribution of channels (Ishihara (1967)) and is supported by Smart (1972, p 1489) who proposed that networks developed under the same environmental conditions would have statistical geometric similarity, that is, "... all corresponding dimensionless variables should have the same distribution functions."
10. The adjustment of channel networks to change of load is discussed by Edwards (1973, p 220) and Chitale (1973, p 293).
11. "Analogous laws of form have been observed to hold in other natural systems as well, such as the branches of a tree" (Smith and Bretherton, 1972, p 1506).
12. 'remuneration of stream segments' to read 'renumeration of stream segments'.
13. 'calculated after remuneration' to read 'calculated after renumeration'.

14. See Appendix C.

15. See Appendix D.

16. The influence of system magnitude could not be removed by determining residuals from regression lines B and C because the difference in slope between the regression lines gives rise to different percentage deviations of E index from the regression line for any system magnitude. Consequently the regression lines must be given a common slope before the residuals can be compared (Wetherill, 1967, p 220).

17. The variable M in Equations 3, 4, 5, 7 and 8 is Shreve's system magnitude.

18. The effects of salt flocculation have been discussed by Meade (1972, p 96-104).